Description for the general public of the project

„Algebraic methods in the problem of approximation of nonlinear control systems"
(Algebraiczne metody w problemie aproksymacji nieliniowych układów sterowalnych)

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The modern control theory is an area which combines classical and the newest methods from various fields of mathematics with applications to concrete problems from different spheres of science and technology such as mechanics, physics, chemistry, biology, economics, sociology etc. This connection is very important; it often occurs that the most natural and perspective basic problems are those which arise in applications.

The control theory was developed as a mathematical theory since the middle of the 20-th century. The first achievements were connected with the problems of control of rockets, aircrafts, spacecrafts as well as with the control of various technology processes. At present, the control theory is applied practically in any area of human activity.

In the control theory, dynamical processes are studied, which are described mathematically mostly as systems of differential or difference equations (under some additional constraints) where some parameters – controls – can be changed at any moment of time during the process, as we wish. The most important problem is to carry out methods for designing controls which transfer a system from a given state to some other target state. This desired control should also satisfy additional requirements: for example, it should be bounded or optimal in a certain sense.

An important feature of the modern control theory is that the studied problems usually are nonlinear and, moreover, their linear approximations hardly can be applied. Hence, linear methods cannot be used, which essentially complicates the solution of such problems and requires development of special nonlinear methods. In particular, this means that, among nonlinear systems, simpler systems should be chosen, in particular, to approximate other nonlinear systems of more complicated structure. In our project we consider nonlinear control systems of a certain form (affine or linear on control and real analytic on state) and study their local behavior. It is important to describe all possible types of such a behavior and, for a given system, to recognize the type of its behavior near a given state. It turns out that this concrete problem leads to basic concepts in pure mathematics such as a free associative graded algebra and a free graded Lie algebra. The classification itself is rather exact: under a natural assumption of non-singularity, all types of local behavior of such systems correspond to graded Lie subalgebras of codimension n (here n is a dimension of the system). Such a classification means that apparently the algebraic tools can be successfully applied, for example, in control design in various situations. In the proposed research we are going to develop new advanced nonlinear methods for analysis of behavior of nonlinear control systems and solving optimal control problems. The obtained results can be used for further theoretical studies as well as for control design in various applications.