

The theory of harmonic functions is one of the most classical areas of mathematical analysis and mathematics itself, studied for over two centuries. Harmonic functions are the prototypical models for solutions of the so-called elliptic partial differential equations and appear in a variety of applications, including the heat flows (temperature distribution), dynamical systems (and related fractals), elasticity theory, quantum mechanics and stochastic processes (stock market analysis). The notion of harmonic functions has been generalized in several ways and in a part of this project we study two of such generalizations. Namely, we will study harmonic functions in the setting of metric measure spaces and define them via the mean-value property. This means that a value of a function at a point equals its integral over a ball divided by a measure of such ball. Metric spaces are spaces where one can measure the distance between two points, e.g. the Euclidean distance, the taxicab distance, the Riemannian distance (appearing in physics). The studies of harmonicity in a generality of metric measure spaces allow us to unify various approaches and compare them to each other in order to extract some essential properties of harmonic functions. Among the topics of the project, let us mention the studies of the boundary behavior for harmonic functions and a question of existence of a harmonic function with the given boundary data. We will also study harmonic functions defined on graphs and trees. Moreover, for some important examples of metric measure spaces appearing in the control theory and cybernetics, the so-called Heisenberg groups and its generalizations, we will investigate our harmonic functions in relation to differential equations.

Another generalization of harmonic functions are harmonic and p -harmonic mappings. These are related to calculus of variations and minimization of certain types of energies defined in terms of the generalized slope of the graph of a function. Apart from interesting theoretical applications, p -harmonic mappings appear in the studies of deformations of elastic bodies, glaciology and in a model of star growth in astrophysics. In the project, we will be interested in answering questions like: how smooth p -harmonic mappings can be? Are there any nontrivial such mappings if the image space of a map is curved? We will be answering these questions largely assuming that the target space is a metric measure space of one of the following two kinds. First type of a space is the one, where we can define triangles made of curves and be able to say that these triangles are thinner than model triangles in the plane. The second type of space is a generalization of Riemannian manifolds whose curvature is bounded from below.

The last part of the project is devoted to studying the mappings generalizing a class of conformal mappings, i.e. transformations which in their classical setting preserve angles between curves. The generalizations we will study may distort balls but only in the controlled manner, that is balls may not be squeezed too much. We will pursue questions of how to recognize, if a map is of that kind by developing a tool used for the same purposes for conformal mappings. The setting for our studies will be the Heisenberg groups.

Our studies will bring a new insight to the notion of harmonicity and develop new tools for studying harmonic functions and their generalizations in various general types of spaces, including important examples of spaces studied in the applied mathematics and physics. The mapping theory answers how the shape of objects and their geometry changes upon applying a map and describes properties of such transformations. Our studies will improve understanding of the mapping theory.