Impact of the mechanisms of the exchange in the nodes on the asymptotic properties of network transport models

Following motto attributed to Albert Einstein that 'Everything should be made as simple as possible, but not simpler', this research deals analytically with efficiency of different methods of description of complex phenomena. A vital problem that appears nowadays is to strike a balance between the simplicity of a method and, simultaneously, the possibility of capturing the key features of a described phenomena. It is natural to assume that the more information is taken into account, the more accurate the description of the situation is given. Unfortunately, there are numerous examples which show that incorporating the whole knowledge of a system into its model results in dynamics that is too complex to allow for any meaningful analysis. Specifying the criteria for an optimal choice between two models of the same phenomena, the transport model on a network and the system of ordinary differential equations, is the main goal of this research.

The most basic example of the transport problem on a graph is the description of one feature of some liquid, changing in time and space, which flows along the network, e.g. sewer system. It turns out that introduction of a different mechanism of the exchange of the substance between the nodes of this network allow for many other applications, then a physical flow e.g. in population dynamics or in queuing systems. In this project I will consider three mechanisms of this kind. Since we fix one direction of the flow, each edge has the beginning and the end and now we can define different mechanisms of an exchange of mass in edges. In one models it is natural to assume that always a constant fraction of mass is shuffled form the end of one edge into the beginning of another. On the other hand, we can introduce an exchange that depends on the total mass at the edge, or one that changes periodically in time.

Let us comment now on the reason why we include the asymptotic considerations. In the modeling of complex networks, there often exists a significant difference between the magnitude of parameters which arises from different time scales of processes in the phenomenon. A simplification of the model can be achieved by exploring this differences. Unfortunately, the dynamics may change if we substitute 0 instead of small parameter and therefore we apply an asymptotic analysis. Results of this kind allow to estimate the error of approximation of more sophisticated models by easier ones. In the case of this study, we expect to obtain a convergence of the system of transport equations to the system of linear, or non-linear, ordinary differential equations. Furthermore, we examine the properties of solutions to the primal problem, and the limit one, such as periodicity of solutions and the superdiffusiveness of the process. They should depend on the boundary conditions and the structure of the network. Better understanding of these processes will allow these models to be a better tool for applications by non-mathematicians.

Lastly, it is worthwhile to mention that besides application for different sciences, this project is important also from the theoretical perspective. It creates connections between two large research fields: partial and ordinary differential equations and, thereby, contributes to the unity of this branch of mathematics.