

Our project concerns modeling of uncertainty in optimization problems, in particular discrete optimization problems. In practice, the input data of optimization problems that support decision making are, almost always, burdened with uncertainty resulting from a lack of knowledge (for instance, the problem is solved first time or an environment changes dynamically), measurement errors, randomness of data, or random events that may have a global impact on the parameters of a problem. For example, in network models of projects (IT, construction ones), which are implemented for the first time. Thus the exact activity duration times are unknown. In such a case the uncertain duration times are often modeled by intervals containing all possible values of activity duration times. Moreover, some global events that are unknown before the project starts (system failure, weather) may also have an impact on activity duration times. It is worth mentioning that when the parameter uncertainty is modeled by closed intervals, which is the simplest way of uncertainty representation, an overwhelming majority of discrete problems, that are polynomially solvable in the deterministic case, become computationally difficult.

The aim of this project is developing various approaches to modeling uncertainty in discrete optimization problems. We plan, among others, an analysis of the impact of uncertainty on computational complexity of the problems. We will focus on the uncertainty representation by means of a *scenario set*, i.e. a set of possible realizations of input data. We will study both single and multistage problems. In the first case, a complete solution has to be implemented before parameter values are set, i.e. before an actual scenario is realized. In the second case, a partial complement or a modification of a solution is possible, after knowing an actual realization of the input data. The multistage problems are much more difficult and challenging from the computational point of view. Depending on the knowledge possessed about a set of scenarios, there exist different criteria for choosing a reasonable solution. If the only information about the input data is a set of scenarios, then *robust approach* is the most popular, in which a solution minimizing a cost in a worst case, i.e. for a worst possible realization of the input data, is found. For this purpose, the minimax criteria are commonly used, which can sometimes lead to very conservative solutions and assume that decision maker is extremely risk-averse. In recent years, several approaches have been proposed that help to reduce the conservativeness of the minimax criteria and take into account the decision maker's attitudes towards a risk. In practical applications the decision maker may have, however, some information about uncertain parameters, for example, he/she can have information about the importance of each scenario. In this case the OWA criterion (*Ordered Weighted Average*) can be applied, which takes into account his/her attitudes towards a risk. If decision maker possesses full or partial knowledge about a probability distribution in a set of scenarios, then stochastic criteria such as *Value at Risk* or *Conditional Value at Risk*, which take into account both probabilistic information in a set of scenarios and his/her attitudes towards a risk can be used to choose a solution. If the exact probability distribution in a set of scenarios is unknown, then one can combine the stochastic approach with the robust one and seek a solution that minimizes stochastic criteria for the worst possible probability distribution. One can also estimate a probability distribution by means of a *possibility distribution* and use the *possibilistic approach* to choose a solution.

In this project, we will perform an in-depth analysis of the approaches mentioned above for choosing solutions for selected discrete optimization problems, that are important for the application of computer science. We will investigate the network, scheduling, production planning and resource allocation problems, assuming the scenario set uncertainty representation of the input data, with or without additional knowledge about the set of scenarios, and in the single or multi-stage setting. Namely, we will explore computational complexity and approximation (approximation ratios, lower bounds on approximation, etc.) of problems under consideration in uncertain environment. This will allow us to understand better the impact of uncertainty on the computational complexity of the problems, which will be an important contribution to the development of optimization under uncertainty. We plan to construct some efficient exact and approximation algorithms (approximation and heuristic ones) – depending on the analysis performed.