## Description for the general public

The notion of *group* is one of the most basic and yet most important concepts in mathematics. It is used to describe symmetries of various objects.

Symmetries are of fundamental importance in our understanding of the Universe, both in macro- and micro-scale. In the first case, the symmetries of time and space are responsible for basic conservation laws, the foundation of modern physics. At the micro level, symmetries of particles determine their chemical features, which, for example, lies at the root of intriguing properties of proteins.

For these reasons, the *group theory*, which is a branch of mathematics investigating properties of groups, that is sets of symmetries, is widely applicable. This project focuses on research in the frame of this theory. It is an essential tool for every mathematician, but it is used also by physicists, chemists, biologists, and in recent years, by computer scientists.

The branch of the group theory, which we work with, is the so-called *geometric group theory* – *GGT* in short. This is a relatively young field, that has developed in the last thirty years. Its questions and tools originate in, among others: combinatorial group theory, algebraic topology, and differential geometry. The GGT studies infinite groups using geometric tools. Roughly, one may say that we are trying to equip a group with a geometric structure. The features of the latter allow us to conclude algebraic properties of the group itself. This strategy is a beautiful example of a situation when two seemingly unrelated approaches to a problem – geometric and algebraic ones – when applied together result in very powerful tools. Indeed, looking at groups as geometric objects allowed to make groundbreaking discoveries about their structures, an example being the case of outer automorphisms of free groups.

The overall objective of our project is to perform precisely such an approach: We want to equip some specific groups with geometric structures allowing us to understand better these groups. Our main tool by means of which we will examine the structures is widely understood, *non-positive curvature*. This concept, which has its origins in differential geometry, is one of basic methods of contemporary GGT, and is a source of its spectacular successes in recent years. It is worth to mention here the solution of the virtual Haken conjecture few years ago. It was achieved by Agol and Wise, by using *non-positively curved cube complexes*.

The first objects that we want to examine using non-positive curvature are *Artin groups*. This is a class of groups known for a long time, studied extensively and still being mysterious. Conjecturally, Artin groups have plenty of nice properties, but no one can prove it. Moreover, there have been many attempts to equip Artin groups with interesting non-positively curved structures. However, this works only in very specific cases. The novelty of our approach is the use of new, combinatorial variants of non-positive curvature. In the past few years we have developed significantly the theory describing these tools and we believe that we are now prepared for using it effectively. Having Artin groups equipped with a combinatorial non-positive curvature we will use it to prove new algebraic, algorithmic, and large scale geometric properties of the groups.

Another objective of the project is the study of interesting topological spaces associated with *Gromov hyperbolic groups* and *Coxeter groups* – important classes of combinatorially non-positively curved groups. The spaces are called *Gromov boundaries* and *CAT(0) boundaries*, and constitute a bridge between group theory, topology, and analysis on metric spaces. At this moment it is not clear what topological spaces can occur here. We want to answer this question.

Furthermore, we plan to use a combinatorial non-positive curvature to provide new constructions of infinite groups with interesting properties. We also want to show that "most" of infinite groups admit a combinatorial non-positive curvature. In other words, we plan to show that being non-positively curved is a "typical behavior" for infinite groups.