In our project we will work on strong cliques in graphs. This problem is associated with strong edges coloring of graphs. Our main goal is to get closer to the solution of a conjecture of Erdös and Nešetřil from 1985 regarding the extremal value of strong chromatic index among graphs with given maximum degree.

A *graph* is a pair of two sets. The first one (denoted by $V(G)$) is the set of *vertices* of a graph, the second one (denoted by $E(G)$) is the set of *edges* of a graph. Each edge contains two distinct vertices. We say that an edge and a vertex are *incident*, if the vertex is one of the two vertices contained in the edge. Two vertices which are connected by some edge are called *neighbors*. The *degree* of a vertex v (denoted by $deg(v)$) is the number od edges incident to v. The *maximum degree* of all vertices of graph are denoted by Δ.

An *edge-coloring* of a graph G is an assignment of natural numbers (called *colors*) to all edges of G. A *strong edge-coloring* of a graph G is an edge-coloring of G in which each edge $e = uv$ must receive a color that does not appear on any edge incident to a neighbor of u or v. A *strong chromatic index* of G(denoted by $\chi'_2(G)$) is the minimum number of colors in a strong edge-coloring of G.

Erdös and Nešetřil gave a conjecture that if G is a graph with maximum degree Δ, then the strong chromatic index of G is at most $\frac{5}{4}\Delta^2$. We want to get closer to the solution of the conjecture by investigating a related problem: determining the maximum size of a strong clique in a graph.

A *strong clique* in a graph G is a set of edges from G such that each two edges are incident to a common vertex or to two neighboring vertices. If F is a strong clique in a graph G, then every edge from F must receive distinct color in a strong edge-coloring of G. Therefore, the strong chromatic index of G is equal to or greater than the size of the maximum strong clique in G - this gives us a way of bounding the strong chromatic index from below.

The strong cliques problem is even more significant if you remember a conjecture of Reed. If the conjecture is true, then any improvement of an upper bound on maximum size of a strong clique in a graph G will give us an improvement of the known upper bound on strong chromatic index of G. This would be an important step towards the solution of the conjecture of Erdös and Nešetřil.

In our project we will also investigate a *distance-t edge-coloring* of a graph G, which is a generalization of a strong edge-coloring of G (for $t = 2$ this is a problem of strong edge-coloring). We will also try to find estimations on the size of maximum *t-strong clique* in G (generalization of strong clique).

We believe that our project will have an impact on future research related to the conjecture of Erdös and Nešetřil. Known results regarding strong and t-strong cliques in graphs are usually by-products of the considerations regarding strong and distance-t edge-coloring. We expect that by putting our focus on bounding the size of strong and t-strong cliques we will obtain stronger and more interesting results.