Periodic solutions of symmetric Hamiltonian systems
(description for the general public)

Physical problems are described by a variety of equations whose solving allows us to predict the behavior of the phenomenon under consideration, to specify its features and to verify experiments we perform. The study, whether given problem **behaves periodically**, i.e. after some time returns to the initial state and behaves just like before, is particularly interesting. Such examples are close to us – the motion of the planets of the Solar System is periodic and, therefore, we observe the seasons, we can predict the solar eclipse for the years ahead and we are able to plan with great accuracy the routs of satellites and spacecrafts. Periodic aspects of motion are part of not only celestial mechanics but also of numerous problems of physics, economy or chemistry.

One of the most important methods to describe the models are **Hamiltonian equations** and Newtonian equations, which can be reformulated to the first one. A great number of such equations possess some symmetries, for example do not depend on the rotation of the whole system or on the change of order of its elements. **The project is devoted to the study of the existence of periodic solutions of Hamiltonian systems with symmetries** in the neighbourhood of stationary solutions, i.e. such conditions for which the system does not change at time. **It will be proven** that under some assumptions we can be sure of the **existence of periodic solutions** of a given problem. Moreover, we can tell when the system returns to the initial state; in other words, we can **approximate the minimal period** of the existing solutions. Let us note that the **hypotheses** we propose significantly **generalize** the well–known theorems due to Berger, Szulkin or Dancer and Rybicki.

The existence of symmetries in equations brings us some extra information, but requires non–standard methods and tools to be investigated. Therefore, one of the parts of the project is the **development and adaptation** of so called *invariant methods* to be applied not only in the problem under investigation but also in a variety of problems equipped with symmetries.

Our motivation to take up the subject of the existence of periodic solutions of symmetric problems were so called **N–body problems**, i.e. the research on the motion of given number of elements (planets, atoms etc.) under the influence of common forces (gravitation, electrostatic force etc.). As the symmetries (the rotations usually) are natural in the equations describing real-life phenomena the inherent part of proof of the theorems will be their **application** to the problems of classical, celestial and quantum mechanics. One of the most important descriptions of such phenomena are **Lennard–Jones equations** which play very important role in molecular physics. The second important example are **Schwarzschild equations** which have a strong influence on the Einstein's theory of relativity and the study of black holes. These problems are the basis of applied part of project but we do not exclude the application of our theorems and methods to the other equations coming from physics.