Description for the general public

The aim of the project is to study equations and systems of semilinear partial differential equations with nonlocal operators and measure data. Roughly speaking, an operator is nonlocal if its value on a given function at a point of the space depends on the values of the function on the whole space, and not, as in the case of local operators, on the values of the function in a neighborhood of the point. As for the term „measure data", we may think that the data of the equation are very irregular.

Semilinear equations with measure data, but with local operators, are subject of intensive investigation for over five past decades. The reason is that they have numerous applications in problems of physics, biology, chemistry and mathematical finance. It is worth mentioning here that linear equations with measure data appeared in the mathematical literature much earlier, as one of the basic tools in the potential theory (the famous Herglotz representation theorem for positive harmonic functions proved in 1911).

Very recently mathematicians started the study of partial differential equations with nonlocal operators, especially with so called fractional Laplace operator. The study of such equations with measure data is rather difficult, because nonlocality of the operator and irregularity of the data result in appearing new subtle phenomena. On the other hand, by the same reasons, new very interesting and quite challenging mathematical problems arise.

In the project I would like to study semilinear equations (both elliptic and parabolic) with measure data involving operators from quite wide class of so-called Dirichlet operators. This class includes all basic local and nonlocal operators. In the study I would like to use probabilistic methods of stochastic analysis and probabilistic potential theory. Since the theory of equations with nonlocal operators and measure data just begin to develop, I would like to focus my attention on some basic problems: existence and uniqueness of solutions, regularity and stability of solutions, large time behaviour of solutions, etc.