

We encounter the Rayleigh–Bénard problem every time we put the kettle on, hoping to boil some water to prepare tea. The water located near the bottom of the kettle is warming up, and, in consequence, its volume increases, and the buoyancy force carries it upwards. On the other hand water located near the surface is cooling, and, as its density grows, moves towards the bottom of the kettle. This phenomenon is known as convection and occurs only if the difference of temperatures between the top and the bottom is sufficiently large. If this difference is small, then water is not moving at all, and if it is sufficiently large the flow will appear to be unordered and restless.

Applications of the Rayleigh–Bénard problem are not limited to the kettle in which we boil water. It is also useful to describe the phenomena taking place in the devices such as cooling systems, heat exchangers, or heat pumps. This problem allows to better understand the processes which take place in the earth's crust or in oceans. Moreover it is also important from purely mathematical point of view, because it makes us closer to understand the phenomenon of turbulence in fluids. The research on the Rayleigh–Bénard model allowed mathematicians to conclude that the number of parameters needed to describe the turbulent flow is finite and even allowed them to distinguish exactly how many parameters are sufficient to describe this particular turbulent flow.

Within this project we will study the heat processes in fluids described by the models similar to the Rayleigh–Bénard one. The aim of the project is twofold. The first aim is the study of the flow of micropolar fluids, i.e., such fluids in which there are immersed small particles which can rotate, and while rotating the extra friction which slows them down appears. The model of micropolar fluid can be used to describe for example the blood flow process in arteries or flow of any fluid which is a suspension of these small particles

Physicists have long supposed, that as there is extra inner friction (of rotating particles) in micropolar fluids, then more energy is dissipated and in consequence the flow of such fluids must be more stable. This means that while in plane water we may see the occurrence of vortices and eddies, the micropolar fluid in same circumstances will be still calm. Within the project we will show the precise mathematical statement of this fact and we will answer the question how much more stable than the plain water is a particular micropolar fluid.

The second topic of the project is Rayleigh–Bénard phenomenon with non-fourier heat law. It turns out that the most well known and widely used equation to model heat propagation leads to paradox which manifests itself in the fact that the small change of temperature in one place after arbitrarily small time reflects arbitrarily far from the location when the temperature changed originally. We replace the most well known model by another one - which bases on the Maxwell–Cattaneo law - and it does not lead to the paradox that the original model had. We will study the model of Rayleigh–Bénard problem with Maxwell–Cattaneo law and we will refer if (qualitatively and quantitatively) to the classical model based on the Fourier law.

The study proposed in this project is a part of the very vivid movement in the theory of equations of mathematical physics, as it allows to construct a relatively simple model of convection in fluids. In our study we will use the newest mathematical methods, and if it is not possible to get the desired result, we will try to derive new methods. For example almost all known to authors of the project results on the Rayleigh–Bénard problem use the maximum principle in the equation for temperature and this principle does not hold in the case of the hyperbolic heat equation. Hence we will need to derive the technique to work with the hyperbolic heat equation to get the counterparts of the results that would normally need the maximum principle.