Description for the general publics

The main aim of our research are solutions of the Schrödinger equation of the form

$$-\Delta u + V(x)u = f(x,u).$$

Solutions of this equation describe the light propagation in photonic crystals. Photonic crystals have periodic optical nanostructure. However, it can have some defects – the periodicity disorder. The external potential V, which appears in the equation, is resposible for the description of the nanostructure, it is of the form

$$V = V_{per} + V_{loc},$$

where V_{per} is a periodic function (responsible for the periodic nanostructure), and V_{loc} is a localized function (vanishing in infinity), which is responsible for the defect. In nonlinear optics, the function f is responsible for the material polarization. The project is focused on the existence of ground state solutions in the presence of sign-changing nonlinearities f of the form

$$f(x,u) = f_1(x,u) - f_2(x,u),$$

eg. $f(x,u) = K(x)|u|^{p-2}u - \Gamma(x)|u|^{q-2}u$. Such nonlinearities allow us to model the phenomenon (depending on the position in the crystal and on properties of the light) of focusing and defocusing of the light in a photonic crystal. We are also interested in the semiclassical limit, i.e. the existence and behaviour of solutions of the equation

$$-\hbar\Delta u + V(x)u = f(x,u),$$

when $\hbar$ is small (more precise: $\hbar \to 0$). This problem makes a connection between the classical (Newtonian) mechanics and the quantum mechanics. Moreover, our research will concern also equations with another operators, which are applicable is different areas of mathematical physics.