

The aim of the following science project is to develop novelty numerical algorithm for solving fractional order ordinary differential equations with many times higher accuracy than its is possible presently and low time computational complexity which is independent of target accuracy thanks to the application of the method of orthogonal collocation points. It employs a functions basis, which has similar shape to the requested solution of a differential equation.

Reasons for choosing the research topic: 1) Singularity in the Riemann-Liouville formulas for fractional order derivatives and integrals decreases drastically calculations accuracy by applying commonly used numerical methods. 2) Non-local character of fractional order derivatives and integrals implies inclusion of function's values from an entire range. 3) Stiff differential equations, which are commonly solved by applying implicit methods, require in solving each step of system of linear or non-linear equations depending on a type of an equation which is solved.

Due to the difficulties presented in points 1-3 obtaining solutions of fractional order differential equations with the help of the computer is difficult. There do not exist any versatile tools for this purpose. On the other hand non-local character of fractional order differentiation and integration operators became an excellent tool for memory and affinity properties description of various physical processes. Reliable sources listed in 2015 over 30 practical applications of fractional calculus, e.g. electrochemical processes formulation, novelty algorithms of image segmentation and control theory. Due to its capabilities, the subject of algorithms for solutions of fractional order differential equations make presently large part of all published scientific papers in the journals devoted to numerical methods.

Differential equations are used for simulations of various physical phenomenon. If a method which is applied for solving them guarantees results of high accuracy and stability, the simulations with its application are more reliable especially if the simulated object is on the verge of stability or simulation's time is long.

The solution proposed in the following scientific project employs: a) application of Gauss-Jacobi Quadrature due to the shape similarity of Jacobi polynomials of a particular degree to a shape of the kernel in the Riemann-Liouville formulas for fractional order derivatives and integrals. It enables to mitigate the influence of the singularity on calculations accuracy and the successful application of the collocation points method, b) application of arbitrary precision for computations enables to eliminate common errors during calculations and increases capabilities of computer arithmetic.

Completion of the aim designated for the scientific project requires: 1) An implementation of new or modify of existing numerical methods for evaluating Jacobi polynomials of any order and their derivatives, finding their zeros and the weights required for the quadrature construction and solving systems of linear or non-linear equations, 2) Computer implementation and joining of selected methods in one program for solving fractional order ordinary differential equations as well as 3) Accuracy and efficiency analysis of all implemented by applying arbitrary precision parts of the algorithm for solving fractional order ordinary differential equations and their practical accuracy and efficiency.