Popular description of the project

a) Characteristic classes of flat bundles

Flat bundles are families of vector spaces over some base (parameter) space which are in some sense locally constant. Globally, however, they can be quite complicated, depending on the monodromy, which is a map from loops in the base to the automorphisms of the fiber. If a loop is deformed continuously, the monodromy does not change. The set of all flat bundles over a given base (also called the representation variety) is an important object in many branches of mathematics and physics. One wants to understand its geometry. "Characteristic classes" are invariants which do not change under a deformation of the flat bundle structure. Thus they help to count the number of connected components of the representation variety. The aim of our project is to provide such characteristic classes, in cases when there are arithmetic restrictions on the fiber, for example when the fibers are vector spaces over an unusual field.

b) Generalized tridiagonal isospectral manifolds

Flag manifolds related to Lie groups play a prominent role in many parts of mathematics, from combinatorics to mechanics (symplectic geometry). They have beautiful topology, which is governed by their symmetry. The aim of our project is to study certain submanifolds of flag manifolds which retain some of their symmetries. Although not as fundamental, their topology and symplectic geometry is very much related to combinatorics. We do expect nice answers and nice pictures, but for this we need to develop the existing methods of computations. These new methods will certainly have applications beyond the special cases considered.

c) Vector interpolation

A space of functions W is k-interpolating if one can find a function in W, which takes on prescribed values at any given k points. Apart form the usefulness in solving IQ tests they have serious applications in data fitting: finding simple rules governing a given set of data. Obviously the smaller W the better: the rule is simpler. Interestingly, the interpolation problem has connections to algebraic geometry. What we want to study is a vector interpolation problem: functions take values in a finite dimensional vector space. A geometric construction of small vector interpolating spaces is possible, and, perhaps surprisingly, gives better answers than the pedestrian approach of converting a vector problem into many scalar problems. The geometric side, related to Grassmann manifolds, is interestingly different from the scalar situation.