Many real life systems (physical, chemical, biological, economical, etc.) can be described using the notion of a dynamical system. In particular, various important processes can be modeled by dynamical systems obtained by iterating polynomials (or, more generally, rational functions). A question of great importance is how a dynamical system behaves with time depending on initial conditions. Typically, the set of possible initial conditions naturally splits into two parts: 1) "regular region" (parameters near which long-time behavior of a system can be accurately predicted) and 2) "chaotic region" (parameters near which long-time behavior is chaotic). In this project I will study complexity of the "chaotic regions" for rational functions. These "chaotic regions" are called Julia sets. I will address the following questions:

1. How large and how geometrically complex can Julia sets be?
2. How hard is it to simulate them numerically?

Below I explain in more details the objectives of the project.

1. In this part of the project I will focus on types of dynamical systems for which Julia sets have more complicated structure. One can expect that more complicated structure of the Julia set means that a randomly chosen initial condition will more likely belong to the Julia set. Thus, system will more likely exhibit an unpredictable behavior. Indeed, there are examples of simple systems with elaborate Julia sets for which if one chooses the starting point of the system at random there is a nonzero chance that the observed behavior will be chaotic. One of the goals of the project is to show that there are various type dynamical systems (generated by polynomials) with very complicated Julia sets of physical size zero. For such systems the regular behavior occurs for almost all initial conditions and, informally speaking, the set of "bad" initial conditions is invisible for a human eye (even though it has a quite complicated structure).

2. Computer assistance play extremely important role in modern science. In particular, many prominent results in dynamical systems were inspired by detailed computer generated images of Julia sets. Computational complexity measures how fast in terms of the number of operations a computer program (or an abstract algorithm) can generate an image of a set within given accuracy. It is known that for the dynamical systems under consideration the computational complexity of Julia sets can be arbitrarily high (i.e. the minimal time required to obtain an accurate images of these regions can be very long). Moreover, there are examples of uncomputable Julia sets. For such examples there is no algorithm which could produce an accurate picture or the set with any given precision.

One of the goals of the project is to show that for most rational functions Julia sets have relatively low (polynomial) computational complexity and thus accurate images of these regions can be obtained using a computer. Another goal is to study computational complexity of some dynamical systems with complicated structure of the Julia sets and design fast algorithms for computing these sets.