Non-commutative probability -
relations between various notions of independence

The notion of independence plays an important role both in everyday life and in mathematics. Intuitive notion of independence of events might be formalized, preserving its basic meaning. The formal definition says, that two events A and B are independent if and only if the probability that both of them occur i.e. the probability of $A \cap B$ equals to the product of probabilities of each of them, so we have

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\,\mathbb{P}(B).$$

The above definition is a foundation of the probability theory.

This project is devoted to the study of independence in non-commutative probability theory. In this theory object that we study are, ulike real numbers, non-commutative with respect to multiplication, i.e. we do not have the property $ab = ba$. It turs out that in such situation there are possible new, natural definitions of independence of non-commutative objects (non-commutative random variables). Among possible notions of independence important role play: free independence, boolean independence, c-freeness and infinitesimal freeness.

Non-commutative probability theory with various types of independence at the first glance might seem very abstract. However this theory has plenty of applications for example in quantum physics, quantum information theory or in wireless networks. It is a consequence of close relations of this theory with random matrix theory.

The aim of this project is the study of relations between notions of independence in non-commutative probability and consequences of this relations for the theory. In particular we would like to focus on relations between free and Boolean independence, the goal of this investigations is better understanding of free infinite divisibility of $*$-distributions. Infinite divisibility is a property which says that for any integer $n \geq 1$, given random variable X might be represented as a sum of n independent, but in some sense same random variables. Our second goal is the study of relations between c-freeness and other structures in free probability. So called c-freeness in some sense unifies the notions of free and boolean independence. It seems to be interesting to find relations of c-freeness and new structures such as infinitesimal freeness. This investigations might result in finding new applications of c-freeness and in particular finding relations between c-freeness and random matrix theory.