Applications of probabilistic methods to some problems of mathematical physics
(a brief general description)

The present project is devoted to the mathematical investigation of some physical phenomena concerning the heat propagation and turbulent transport, with the methods that are rooted in the theory of probability and stochastic processes. Due to the high complexity of the systems we have in mind (crystals and fluids made of molecules, or atoms, or a turbulent flow of a fluid) it is quite natural to describe their evolution using models that involve stochastic processes, or random fields. Frequently the interest in resolving such models stems from the fact that one wishes to obtain a macroscopic description of the behavior of the system, while the model itself is formulated in the microscopic scales. To achieve this goal one may avail oneself of powerful techniques of the theory of probability and stochastic processes.

Our present project consists of two parts. In the first one, we propose to investigate the problem of heat transport in a one dimensional medium. In this case the heat conductor is described as a chain of harmonic oscillators with a random mechanism of momenta exchange whose dynamics is governed by some stochastic process. We would like to investigate finite systems with boundary points attached to thermostats (that maintain fixed temperatures at the endpoints). An interesting feature appearing in the investigation of such systems is the fact that the limiting equation for the heat energy transport takes form of a fractional diffusion equation. In addition, in this part, we plan to investigate the problem of the thermal vs mechanical energy evolution for chains of oscillators. They refer to two forms of energy of vibrating molecules, those that possess a macroscopic profile (more structurized), called *mechanical energy*, and random fluctuations around it that we identify with the *temperature* (or thermal) profile. Our aim is to investigate the macroscopic evolution of the thermal vs mechanical energies in the so called non-acoustic chains (i.e. those with the speed of sound equal to 0).

In the second part of this project, we plan to investigate the problem of the motion of a particle, sometimes also called a *tracer*, in a random velocity field. It has been a long standing hypothesis, supported by numerical experiments, that if a particle moves in a sufficiently disordered random field, its trajectory satisfies the central limit theorem, i.e. its motion resembles that of a Brownian particle. This phenomenon, if holds, is called *a turbulent diffusion*.

The author of the project has already spent some time in the past investigating this topic and obtained some partial results towards proving that statement. With a recent progress achieved in the ergodic theory of stochastic processes, especially due to an application of coupling techniques, we are hoping to obtain a significant strengthening of our former results. The second topic concerning the problem of motions in random fields deals with the issue of spatial stationarity of the flow. The latter means that the statistical description of the flow does not change under the temporal and spatial translations. Most of the results obtained so far have required this assumption. Here we propose a notion of the local ergodicity that allows us to locally average tracer motion and obtain a diffusive approximation for the trajectory. Finally, we plan to consider locally ergodic environments that are not rapidly, but only moderatly decorrelating. In that case we simplify the situation by assuming that the velocity field is only slowly varying in space and is Gaussian. In our present proposal we wish to use the rough path techniques to prove the superdiffusive character of the tracer motion in that situation.