In this project we want to work on online coloring and online $L(2,1)$-labelings of geometricly defined graphs. The topic of geometric grapgs is intesively studied for many years, both beceause of theoretical connections and practical motivations. In particulr, some of the coloring problems from this project can be related to real problems. For example, frequency assignement and time slots assignnement in telecommunication networks.

A *graph* is a pair of two sets: the first one is the set of *vertices*, while the second contains pairs of distinct vertices, and each such pair is called an *edge*. A pair of vertices is called adjacent if they form an edge in the graph. One of the classic problems in graph theory is called graph coloring. The goal is to assign each vertex a color from fixed set, so that two adjacent vertices receive different colors. We want to use as small number of colors as possible. This project concerns *intersection graphs* of geometrical figures. They are defined in the following way: the set of vertices is the set of figures, and two vertices are adjacent if the corresponding figures intersect. In this project we will concentrate on two such classes: intersection graphs of intervals in the set of real numbers and intersection graphs of disks on the plane.

To make it clear, let us explain how such graphs can model conflicts a a telecommunication network. If we assume that the disks show areas in the ranges of individual transmitters, the nonempty intersection of two discs means that, for example, we should give these transmitters other frequencies (that is, different colors). Different types of geometrically defined graphs and types of coloring may include various properties and requirements of the various types of telecommunication networks. For example, $L(2,1)$-labeling requires that neighbors have to get labels differing by at least 2 and two vertices withs a common neighbour. Such a model can be useful if we consider that close frequencies can interfere (hence we 'strongly' distinguish neighbors) and a common neighbor of two transmitters implies that those transmitters have to get different labels. In some such cases, the whole structure is not known in advance, but is revealed in steps (for example, telecommunication can be gradually expanded). Such problems are called *online* and such versions of coloring and labeling are form the majority of the research tasks. Online coloring means that the graph is not known at the beginning. Then in each round one vertex is revealed with all of the edges to previously revealed vertices. This new vertex has to be colored right after its presentation. What is important, a choice cannot be changed later, hence the situation is more difficult.

In our work we want to create better coloring algorithms (thath is, recipes) for such graphs, but also theoretically constructed bound on the performance. Naturally, the key to success is to use geometrical properties of disk intersection graphs that make them unique in the class of all graphs. Along with the attempt to understand those properties deeply, we will also work on certain Euclidean plane coloring problems related to the classic Hadwiger-Nelson problem. The question in this problem is: How many different colors suffice to color all points of the plane so that each pair of points in the distance exactly one have different colors? It is known that it can be done using 7 colors with relatively simple pattern and it is impossible with less than 4 colors - the minimal number is not known. It appears that such questions are not only interesting from mathematical point of view, but also they can be a tool to design better algorithms in the aforementioned cases of intersection graphs.

The algorithms, bounds and coloring schemes created within this project are expected to not only give understanding and solutions in particular problems. A nonformal goal is that they will help in developing techniques and new ideas, which will find broad applications in constrution of coloring algorithms of geometrical objects. As a consequence, the project may bring benefit both in the theoretical area and in the related real problems.