Homology groups of configuration spaces for particles on graphs

A graph is a set of points, called vertices, along with edges that connect some of the vertices. An example of a graph that is used by thousands of people on a daily basis is the underground - the vertices of the graph are the stations, and the edges are the tracks that connect the stations. There are also graphs that are too small to be seen with the naked eye, which are fabricated from nano-wires no more than a few millionths of a millimeter in diameter. The objects that inhabit these *nano-graphs* are particles so small that their motion is governed by the laws of quantum mechanics rather than classical Newtonian physics.

Suppose we put some number of objects on a graph; the objects could be trains or particles, depending on the context. We can place each object anywhere we like, provided no two objects are put in the same place. Such an arrangement is called an *allowed configuration*, and the object of our research is the set of all such allowed configurations of objects on a graph. Next, we imagine that the objects can move; they can move along the edges of the graph, and at a vertex, they can move to a different edge. However, since two objects can't be at the same place at the same time, they cannot pass through each other – the objects never collide.

Our research problem concerns quantum particles moving on a nano-graph. An important fact about these particles is that they are indistinguishable; there is no way experimentally to tell them apart. From the point of view of configurations, this means that we only care whether or not there is a particle at a particular place on the graph, and not which particle is there. But from the point of view of quantum mechanics, indistinguishability turns out to have profound consequences. When quantum mechanics is applied to particles moving in our three-dimensional world, the fact that the particles are indistinguishable vastly restricts their possible collective behaviour to just two types, called *fermions* and *bosons* (and which of these types applies is determined by another quantum mechanical property of the particles, called *spin*). For example, electrons are fermions, and this fact is responsible for a fundamental law called the *Pauli Exclusion Principle*, which in turn underpins our understanding of the periodic table of elements. The two alternatives taken together – fermions or bosons – are called *quantum statistics.*

An important and surprising discovery in physics in the last 50 years is that if particles are constrained to move in two rather than three dimensions, they can in principle exhibit new forms of quantum statistics, called *anyons.* Our recent research has shown that particles constrained to move on a graph can exhibit even more exotic forms of quantum statistics, depending on the topology of the graph. In fact, our previous work was based on the simplest topological information about many-particles configurations on a graph. Our proposed research is to answer the question, whether any new forms of quantum statistics may arise when we take into account more complex topological information, called *higher homology groups*, which may also be associated with configurations of particles on graphs. We are particularly interested in some specific part of higher homology groups, which is called the *torsion.* Our current methods allow us to study the simplest case, where we can hope for the existence of new quantum statistics, which is the third homology group.