DESCRIPTION FOR THE GENERAL PUBLIC

The project consists of three research topics at the crossroads of number theory and dynamical systems, a vibrant area of research without clearly determined boundaries. It includes such topics as arithmetic dynamics over number fields and local fields, ergodic Ramsey theory, moduli spaces of dynamical systems, homogenous dynamics, arithmetic codings (including β-expansions), iterating power series over finite fields and local fields, and even problems such as the Collatz conjecture.

The main objective of the research project is the development of new number theoretic and algebraic techniques required to study problems of dynamical origin. In the project, we will study three research topics:

A. Dynamical zeta functions of rational maps in positive characteristic.
B. Deformations of local group actions and automatic sequences.
C. Higher dimensional analogues of the Davenport–Erdős Theorem.

The three topics have their origins in different domains. The first topic investigates rationality and transcendence of dynamical zeta functions of regular maps of the projective line over a positive characteristic base field. It belongs to the relatively recent domain of arithmetic dynamics, but its origins are strongly related with complex dynamics. The problem has been completely solved in characteristic zero. In positive characteristic, however, there are only some recent results of Bridy which are very restrictive in their scope.

The second topic proposes a new approach to the widely investigated problem of classifying deformations of local group actions. We intend to use techniques related to automatic sequences to study them. Automatic sequences belong to a subject of origin in theoretical computer science, but strongly related to algebra, symbolic dynamics, logic, and other branches of mathematics.

The third topic is concerned with combinatorial properties of sets of B-free numbers. For a subset B of the positive integers, the set of B-free numbers consists of integers not divisible by any element in B. These sets were first studied in 1930s in relation to abundant numbers. The results of Davenport–Erdős became recently important from a dynamical point of view. In 2010, Sarnak proposed to study the square-free subshift and expressed his hope that a more careful study of dynamical and ergodic properties of this system might lead to developing techniques that would lead to better understanding of prime numbers distribution. Similar questions can also be posed for the sets of B-free numbers. Some work has been done in the recent years on extending these results to shifts with an action of a group $\mathbf{Z}^d$ with $d \geq 2$. A major obstacle that makes this problem seem considerably more difficult than its one-dimensional analogue is the lack of a higher dimensional version of the Davenport-Erdős Theorem that would apply to B-free sets of lattice points.

In order to solve the problems described above, we will certainly need a wide range of methods from domains such as algebraic and analytic number theory, non-archimedean analysis, algebra, automatic sequences, arithmetic geometry, and dynamics.