Nonlocal parabolic problems: regularity, blowup, pattern formation

This project is aimed at studies on problems of the existence, regularity and stability of solutions of models from biology and physics of continuous media involving nonlocal diffusion operators as well as nonlocal, nonlinear interactions (described by e.g. *mean field* approximation).

The first group of models include **Chemotaxis and other quasilinear parabolic problems**. The fundamental questions to be answered are related to local- and global-in-time solvability of various chemotaxis models generalizing the simplest classical Keller–Segel model (*e.g.* by replacing the usual Brownian diffusion by a Lévy diffusion) with possibly singular data and analysis of asymptotic profiles of blowing up solutions.

The second research task concerns **Nonlocal porous media equation in bounded domains** when the pressure is described by nonlocal operators acting on the density and the rigorous analysis of physically relevant boundary conditions is particularly delicate. We intend to study existence of solutions, local regularity properties, qualitative properties such as speed of propagation, finite time extinction property, and their long time asymptotics.

The third group of problems is related to **Reaction-diffusion-ODEs in models of pattern formation** applicable for a description of interacting populations and multiscale cells complexes (such as in processes of angiogenesis and early stages of carcinogenesis). Here, diversification of cells, biochemical and purely mechanical (cell packing) effects can be taken into account. We will study mechanisms of formation of singularities by destabilization of steady states, and classify solutions forming patterns in infinite time.

Significance.

The above mentioned particular questions are closely related to each other by the use of planned methods of analysis, and fall into a general plan of studies of solvability of models with singular data, continuation of solutions and the determination of their asymptotics (both in time and in space), and formation of singularities eventually leading to spatial patterns and/or blowup of solutions.

The ultimate goal of study is to understand the conditions of formation of new structures, fixed or moving singularities and behaviours similar to phase transitions in statistical mechanics. This could provide explanations of symmetry breaking and pattern formation in biology and medicine (such as growth of a biological tissue), in shapes of animal coat markings, and in oscillating chemical reactions.