Morse theoretical methods in Hamiltonian dynamics

Hamiltonian equations describe evolution of physical systems coming for example from classical mechanics or electrodynamics. For a complicated energy function it is often very difficult to give a precise solution of a given system. One possible way to overcome this problem is to use topological methods on infinite-dimensional spaces. In particular, such an approach is being effectively used to search for periodic solutions, i.e. solutions which come back to the initial state after a fixed time. A more difficult question is that about the existence of homoclinic orbits. A homoclinic orbit is a solution which both in a positive and negative time tends to a stationary state, i.e. a critical point which is a rest point of the physical system. Let us emphasize that the study of existence and multiplicity of homoclinic orbits is one of the main problems in the theory of Hamiltonian systems. Their existence may imply Smale's chaos in discrete dynamics of the system (see the horseshoe map). While the existence of infinitely many homoclinics indicates that the system is not integrable. Another interesting question is whether the assumption that the system is periodic in time can be changed to be almost periodic. The concept of almost periodic functions appeared at the end of the last century in the papers of P.H. Rabinowitz. It is related with celestial mechanics. Actually, movements of planets are usually almost periodic.

In this project we would like to modify known research methods in such a way that they can be applied to the open problems, in particular, to Hamiltonian systems with a non-standard kinetic energy or Hamiltonian systems with a standard kinetic energy but with a complicated configuration space. We will be concerned with topological methods such as infinite-dimensional versions of Morse homology, Floer homology, Conley index and variational methods if it is possible to define an underlying action functional.

Since Marston Morse published his works on geodesics, his methods became one of the main tools in investigating Hamiltonian systems. The modern Morse theory is based on various variational principles designed for finding critical points of the action functionals, the two most important of which are Lagrangian and Hamiltonian action functionals. While the former functional can be efficiently studied by the infinite-dimensional generalization of Morse theory which Palais and Smale elaborated in the 60’s, the study of the latter one requires more sophisticated methods. Among these, the theory which Floer developed in the late 80’s.

It seems also to be interesting to find relations of Morse homology with other topological invariants such as Floer homology or Conley index. In the past, the study of relations between different Floer homology groups led to important results in differential and symplectic geometry.

The Polish-German project will be hold at the Faculty of Applied Physics and Mathematics of Gdansk University of Technology and at the Faculty of Mathematics of Ruhr-University of Bochum under the supervision of dr hab. Joanna Janczewska and prof. dr. Albero Abbondandolo.

At the end it is worth pointing out that the father of Floer homology, Andreas Floer studied at Ruhr-University of Bochum, where in 1982 he got a master degree and in 1984 he defended his PhD thesis. Since 1990 till his tragic death in 1991 he worked there as a professor.