DESCRIPTION FOR THE GENERAL PUBLIC (IN ENGLISH)

Fourier transform is a basic tool which allows us to analyse a signal in a dual manner – by studying it in the time domain (or in the space domain in case of images), where it is represented by the amplitudes of samples (e.g. values of pixels), or by investigating it in the frequency domain, where the signal can be represented by the (infinite) sums of cosine and sine functions, each with different frequency and amplitude.

The classical signal theory deals with real- or complex-valued signals (time series or images). However, in some applications described in literature signals are represented by more abstract structures, e.g. elements of hypercomplex algebras (generalizations of real and complex numbers). The main object of our research is the octonion algebra, i.e. numbers of the form

$$o = x_0 + x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3 + x_4\mathbf{e}_4 + x_5\mathbf{e}_5 + x_6\mathbf{e}_6 + x_7\mathbf{e}_7, \qquad x_0, \ldots, x_7 \in \mathbb{R},$$

where there are seven different imaginary units (each with the property $\mathbf{e}_i^2 = -1$). Multiplication in the octonion algebra is in general not only non-commutative, but moreover it is not associative, which makes the extension of results known for the real and complex spaces much more difficult.

So far, practical applications of octonions have been limited to mathematical physics and theoretical studies of other algebraic structures. However, recently there have been published results of the research concerning hypercomplex analytic functions, where the concept of octonion Fourier transform was applied. It is a generalization of the classical tool of the signal theory, well studied in the real and complex case. So far the literature of the subject lacked the theoretical considerations about the well-posedness of the definition and about the properties of the transform. We were able to make a first step in this direction by showing that the octonion Fourier transform of a scalar (real-valued) function of three variables is well defined and satisfies some basic properties.

We are convinced that there is a need of further development of the above-mentioned theory, e.g. verifying that the octonion Fourier transform definition is valid in the case of octonion-valued signals, deriving other properties important from the point of view of applications in signal processing, or extending the octonion-based signal theory to the case of discrete variables. Due to the lack of elementary properties of the octonion multiplication, studying those issues is very complicated and error-prone. Considering that, we aim to develop a tool for automatic (symbolic and numerical) computations, i.e. a Mathematica package and a MATLAB toolbox which would simplify our research. The above-mentioned directions will be the main part of our project.

It is worth mentioning the benefits of the eventual success of our research. In the literature one can find examples of applications of the quaternion Fourier tranform in the analysis of some time-invariant linear systems given by partial differential equations of functions of two variables. They show the prevalence of the hypercomplex approach over the classical theory – it is now possible to study each time-like axis of the solutions of those equations independently. Analogous results for the octonion Fourier transform would allow us to study each of the three time-like axes of the solutions of some partial differential equations independently. It will open the way to some practical applications of this theory.