SUMMARY

We plan to explore problems that involve interactions between logic (descriptive set theory, as well as model theory), Ramsey theory, topological dynamics, and continuum theory. The project is divided into three parts:

- automatic continuity of homeomorphism groups of compact metric spaces;
- universal minimal flows of homeomorphism groups and Ramsey theory;
- groups of measurable functions.

This project focuses on algebraic and topological properties and on dynamics of second countable completely metrizable topological groups, which are called *Polish groups*. This class includes locally compact groups (in particular, Lie groups), but there are many more examples. Polish groups come up naturally in descriptive set theory, model theory, theoretical computer science, Ramsey theory, topological dynamics, continuum theory, ergodic theory. An important class of Polish groups are *permutation groups*, that is, automorphism groups of countable structures. Other examples are among isometry groups of metric spaces, or homeomorphism groups of compact metric spaces.

There is a number of phenomena that occur for some Polish groups and do not occur for locally compact groups. For example, every locally compact and not compact topological group always has continuum many non-isomorphic minimal flows, and always admits non-metrizable ones. On the other hand, there are many Polish groups which have only one minimal flow equal to a single point (for example, the group of all unitary operators of the infinite dimensional separable Hilbert space with the strong operator topology is such).

In the large part of the project we want to focus on homeomorphism groups of compact metric spaces which are (approximately projectively) ultrahomogeneous. A prototype of an ultrahomogeneous structure is the set of rational numbers equipped with the usual order: any order preserving bijection between two tuples (of an arbitrary length) of rationals can be extended to an order preserving bijection of the whole set of rationals. There is a number of ultrahomogeneous structures among graphs, hypergraphs, or metric spaces. A structure is *ultrahomogeneous* if any isomorphism between finite substructures of it extends to an automorphism of the whole structure. An important example of an ultrahomogeneous structure is the *Urysohn metric space* U, that is, the unique up to isometry complete separable metric space such that every finite metric space embeds isometrically into U and every isometry between two finite subspaces of U extends to an isometry of U. Isometry groups and homeomorphism groups of (approximately, projectively) ultrahomogeneous structures are always very rich, which makes studying them very non-trivial and fascinating.