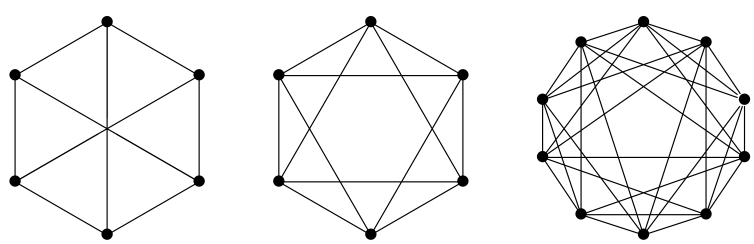
Algebraic Torus Actions: Geometry and Combinatorics
Description for General Public

Algebraic geometry is a central branch of modern mathematics. Its roots are in the classical and differential geometry, analysis, topology, and commutative algebra. The research in algebraic geometry is strongly motivated by its vital applications in other branches of mathematics, theoretical physics, computational biology, computer science and engineering.

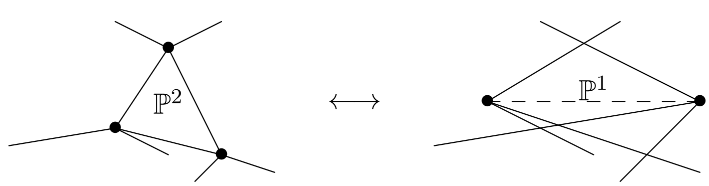
On the other hand, algebraic geometry applies plenty of methods stemming, among others, from topology, analysis and discrete mathematics. Symmetries of algebraic and geometric objects, which are known as *varieties*, allow to apply methods from combinatorics. Because of symmetries, or more generally, because of *group actions* one can reduce problems concerning complex multi-dimensional varieties to, possibly easier, questions regarding discrete objects or polytopes.

The present project is focused on varieties which admit algebraic torus action. It is essential that many properties of such varieties can be encoded in terms of objects which live in the usual Euclidean space. Following this idea, we introduce the notion of a *grid* of a variety which is a structure consisting of points and line segments in a Euclidean space. The grid encodes information about both the variety and the algebraic torus action.

As an illustration, below we present grids of varieties with two dimensional torus action so that the grids can be drawn on the plane. The left-hand-side diagram is the grid of the space of flags, i.e. pairs: point and a line containing it, in a (projective) plane. The central grid is of 4-dimensional quadric while the right-hand-side grid encodes 6-dimensional grassmanian, which is the space of lines in a 4-dimensional (projective) space.



Using grids we expect to understand, among other things, small modifications or surgeries of varieties, called *flips*. Below, we present a modification of grids which corresponds to a flip of a 4-dimensional variety. As result of the flip a plane $\mathbb{P}^2$ is replaced by a line $\mathbb{P}^1$.



Finally, let us note that the notion of a grid, which we described above, is only one of many tools which we intend to use while studying varieties in terms of discrete and combinatorial objects associated to the algebraic torus action.