

## DESCRIPTION FOR THE GENERAL PUBLIC

### Algebraic methods in the study of the quantum Yang-Baxter equation - braces and associative algebras

The project is concerned with algebraic methods and algebraic structures created in the study of the quantum Yang-Baxter equation. This equation has laid foundations for a rapid development of a number of areas of mathematics, and especially the theory of quantum groups and related aspects of the theory of Hopf algebras. The quest for solutions of this equation, as well as its several applications, has been accomplished to a large extent via developing algebraic tools, based on the theory of associative algebras, theory of groups and semigroups and associated combinatorial and algebraic structures. The fundamental open problem motivating this project is concerned with the description and classification of the so called set theoretic solutions of the quantum Yang-Baxter equation. The proposed methods are based on algebraic structures called braces and on certain special classes of associative algebras, including algebras related to the so called braid relation, that has played an eminent role in several areas of mathematics.

It is known that a description and classification of all set theoretic solutions of the quantum Yang-Baxter equation can be reduced to a description of all braces. Therefore, the first fundamental problem is to describe all finite simple braces (as the classification of all finite simple groups was a fundamental problem in the theory of finite groups). And, in the next step, the natural basic question is: how can one build all braces from these simple structures ('extension problem')? Secondly, the project assumes a study of combinatorial and structural properties, as well as representations, of certain classes of associative algebras. On one hand, these include algebras associated in a natural way to braces and to the braid relation. On the other hand, algebras satisfying the so called pentagon relation, that have shown up recently in several mathematical contexts (representation theory, algebraic combinatorics, combinatorial theory of associative algebras, mathematical physics). The aim is to study and to understand relations between these areas, hidden in the nature of the considered types of relations that are used in the presentations of these algebras. And with a view towards applications in the context of the Yang-Baxter equation.

The motivation and the starting point for this project come from recent results of many authors, that are on the crossroads of the theory of quantum groups, representation theory, structural and combinatorial ring theory, group theory, theory of Hopf algebras, knot theory and some aspects of mathematical physics. The fact that this is an area of current interest is, in particular, reflected by several recent papers in leading mathematical journals, as well as around 50 preprints related to the Yang-Baxter equation posted in the archive <http://front.math.ucdavis.edu/>, in section 'mathematics', only in the years 2015-2016.