Value-distribution of zeta- and L-functions

One of the most important branch of modern number theory is the so-called analytic number theory. Its main objective is application of analytic properties of the so-called complex zeta- and L-functions to research concerning certain arithmetic, algebraic and geometric objects. These functions, as important generalizations of the classical Riemann zeta-function, are generating functions formed out of local data associated with either arithmetic or algebraic objects. An important research problem is the distribution of values taken by zeta- or L-functions on any vertical line lying in the so-called critical strip. This subject requires to use advanced and subtle methods of the theory of analytic functions and classical number theory. The better understanding of value-distribution allows to apply more advanced techniques from the theory of analytic functions and might give new results concerning basic arithmetic and algebraic objects.

It is well known that values of zeta- or L-functions acts quite chaotic in the critical strip. For example, one can show that the Riemann zeta-function and its reciprocal take large values, the so-called extreme values, on every vertical line in the critical strip. It is important to give lower bounds for such extreme values. This subject is well-investigated for the Riemann zeta-functions, mainly thank to recent results from the last decade. However, known results concerning the case of general L-function are still far from what is expected by experts. So one of the main research objectives of project deals with some improvements of known lower bounds of extreme values of L-functions and characterization of points where these extreme values are taken.

Another important evidence for irregularity of value-distribution is the so-called universality property. Roughly speaking, it states that any analytic landscape can be found (up to an arbitrarily small error) in the analytic landscape of the Riemann zeta-function. It certifies the richness of values of the Riemann zeta-function and uniqueness of this type of functions in the space of analytic functions. Now, it is known that there exists a rich zoo of universal Dirichlet series, but still there are many difficult unsolved problem in the theory of universality. One of them concerns an effective version of universality theorem, which is still unknown in general setting. The important step into this direction and simultaneously another objective of the project is to investigate and characterize a density of the set of parameters t such that a given analytic function can be approximated by certain purely imaginary shifts of the Riemann zeta-function. Another aim of the project deals with the so-called joint universality, which, roughly speaking, states that any analytic functions can be approximated simultaneously by the same imaginary shifts of a collection of L-functions. One of the main task in the project is to find a new type of joint universality theorem. Theorems of this type provide new information related to independence in the set of L-functions.

All research tasks are planned to achieve by applying modern methods of analytic number theory. Research topics of the project play a central role in number theorists' interests. New results will allow to better understand the phenomenon of zeta- and L-functions, and their analytic properties.