Pointwise regularity theory for sets, measures, and varifolds

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Imagine a circle made of a flexible wire. Now bend the wire into some fancy shape (make a knot out of it but do not pull it tight) and dip it into a soap solution. Pull it out and you will get a soap film (a *surface*) bounded (*spanned*) by the wire. One can easily create such soap films with self-intersections, which we call *sigularities*. Theoretically there are many other types of singularities possible. In general we call a point on a surface *singular* if smaller and smaller neighbourhoods of that point on the surface do not get closer and closer to some two dimensional plane (the *tangent plane*) after rescaling.

Mathematicians model soap films using so called *minimal surfaces*, i.e., surfaces having mean curvature zero. We also consider such "surfaces" of dimension m other than 2 and lying in a space (Euclidean or not; e.g., the universe (i.e. Einsten's spacetime) is not Euclidean) of dimensiona n other than 3 – then the boundary "wire" is of course of dimension $m - 1$.

Minimal surfaces of dimension 1 are just shortest paths (*geodesics*) connecting two points (in this case the boundary is of dimension $m - 1 = 0$). Two dimensional minimal surfaces turn out to be useful for architects who like to build fancy non-flat building but have to minimise the tension inside walls and roofs. In general relativity these surfaces also play an important role for studying the universe.

We say that a surface is *regular* if it does not have any singularities. Many regularity questions concerning minimal surfaces remain unanswered. How often can the singularities occur? What kind of singularities can happen? Our project aims to give partial answers to these questions. Moreover, we will study not only minimal surfaces (mean curvature zero) but also other well behaved surfaces (mean curvature does not change too fast) and we will develop new mathematical tools for dealing with quite arbitrary subsets of an Euclidean space.