Popular science description

Because of the wide range of possible applications, models which are used in the theory of anomalous diffusion are varied. They make use of large number of methods and notions from the theory of integral and differential equations, stochastic processes and statistics. Classical and generalised Langevin equation ha a special position among these models, which is caused by reasons related both to physics and mathematics. For a physicist this equation links the theory of anomalous diffusion with classical mechanics and statistical physics, because it is strictly derived from these theories. From the mathematical point of view, it has a rich structure, which allows to interlink notions from different branches of theory of stochastic processes and integro-differential equations. This elasticity also influences the practical usage of the Langevin equation in different areas, such as biology, electrical circuit theory and financial engineering.

The goal of the project is connected to the reasons of the Langevin equation's popularity. We plan to analyse a class of models based on the Langevin equation, which use the notions of generalised derivatives, described by the theory of the fractional derivatives, and processes studied in the theory of anomalous diffusion, such as random walks.

We have chosen this class of models, because it opens new range of applications and the possibility to make use of the latest mathematical results in the considered areas. One important reason for proposing this project is that it the studied models are mathematically rich and at the same time important from the applicational standpoint. Therefore the planned research will concentrate on following the relations between the parameters of the equation and properties of its solutions, which has a crucial applicational significance, but is also an interesting theoretical question, which allows to study non-typical relations between mathematical objects, which do not appear often outside this particular context.

As an addition to the used mathematical methods we want to perform numerical simulations and statistical analysis, which will additionally verify the obtained results. At the same time such study will present how to model a real life phenomena with the obtained results which should be an important step toward the future usage and later development of the considered models. We want to spread the subject of Langevin equation as an applicational model and also in the other areas related to the project, such as the theory of the random walks and fractional derivatives. It is also one of the important goals of the project, as well as our additional motivation.

This interdisciplinary research project should contribute significantly to the better understanding of the anomalous diffusions and fractional dynamics. The theoretical results related to generalized Langevin equations can help to unify the mathematical description of anomalous dynamics and to find its proper physical interpretation, whereas the applicational part can help to build statistical methods appropriate for the analysis, verification and estimation of the parameters of anomalous complex systems. These up to date issues, which are at the moment the subject of studies of various leading scientific centers (Cambridge, Heidelberg, MIT, Princeton, Tel Aviv, München), will have considerable impact on mathematical, physical and biological sciences.