DESCRIPTION FOR THE GENERAL PUBLIC

The goal of the project is to study and utilize the interaction between two important areas of pure mathematics: ergodic theory and topological dynamics. These two fields exhibit deep connections and we plan to use ideas and techniques from ergodic theory in order to solve problems in topological dynamics and vice versa.

Topological dynamics and ergodic theory are subfields of the more general field of dynamical systems. Examples of dynamical systems are the solar system, the weather, the production of white cells in the blood, the motion of billiard balls on a billiard table, the movement of gas molecules in a container, sugar dissolving in a cup of coffee, the stock market, the formation of traffic jams etc. etc. Although these examples and numerous others in the fields of physics, biology, chemistry, engineering, mathematics, ecology... may differ substantially from each other, they can all be modeled by a *phase space* and an *evolution rule*. The phase space consists of all possible *world-states* (e.g. the locations and velocities of the planets of the solar system) whereas the evolution rule is the transformation which sends every world-state, representing the state of the system "now", to the world-state representing the state of the system "one unit time later".

Any modeling of a dynamical system consists of keeping some information and discarding some other information, for example, is a gas molecule modeled as a point or a ball or a more complicated structure? In topological dynamics the information retained is the "closeness" of world-states, specified for instance, by a distance function $d(x,y)$ between all pairs x,y of worlds-states. In ergodic theory the information retained is the relative probability of different world-states. Discarding some of the information describing real life systems enable mathematicians see the "big picture" and discover unifying principles.

Let us illustrate two of the main problems the project aims to tackle. For the sake of clarity we will only give a simplified outline that conveys the main ideas.

Imagine an airport which communicates with incoming and outgoing planes through radio communication in a certain band of frequencies. According to the regulations no one else is allowed to use this band in order to ensure safe takeoffs and landings. However some pirate radio stations are consistently violating the law and broadcast in this band in changing frequencies. We would like to model this problem with the help of topological dynamics and find conditions which guarantee, at least theoretically, that one can locate efficiently the frequencies used by the pirates. Previously some scientists proposed a probabilistic model which ensures 99% success but not 100%. Relying on the deep connections between ergodic theory and topological dynamics, we aim to improve the existing model, achieve 100% success but at the same time sacrifice efficiency as little as possible.

The second problem which the project aims to tackle is to show that certain averages for ergodic systems, the so-called *nonconventional ergodic averages*, converge, that is have a limit. Going the opposite direction from the previous example we aim to use topological dynamics as a tool for the solution. Specifically we would like to model the ergodic systems by a certain topological system with good properties.

To summarize, ergodic theory and topological dynamics are two important subject of pure mathematics which exhibit deep connections. By providing new links between these areas the project will enhance the understanding of both subjects.