

The proposed research aims not only at solving very hard problems in mathematics; it will also advance the field of computer-aided mathematics, also known as *validated numerics*. Here, we are not simply using the computer to produce numerical approximations (of unknown quality) of a dynamical system; we are using the computer to *verify* mathematical properties of the system.

When it comes to continuous problems like the ones we are addressing, this is a true shift in paradigm: we are now entering an era when very advanced numerical computations can come equipped with a mathematical certification. All necessary error bounds are done on-the-fly along with the computation itself. This additional information can be used to adaptively modify the computations as they are performed: the level of discretization of the problem can be fully controlled, and locally adapted to the problem. This fundamental change results in efficient and extremely fault-tolerant numerical methods, ideal for the systematic study of nonlinear systems.

It is our belief that validated numerics will play an instrumental role as computers become increasingly dominant in every-day life, as well as in scientific research. The more we rely on numerical computations, the more we must be able to trust them. This is important in many applications such as control of industrial robots, and in the basic research stages of drug development. And from the mathematician's point of view, the ability to use numerical computations as an integral part of a proof opens up a huge range of problems that have previously been out of reach.

The theories and techniques of validated numerics define a new research area, along with a new breed of scientists equipped with the combined strengths of pure mathematics and scientific computing.

Our aim is to apply this approach to PDEs. One of the seven Millennium Problems concerns PDEs, in particular the Navier-Stokes equations, which are related to our investigations. Quoting the Clay Mathematics Institute, who are the founders of the Millennium prize: 'Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier-Stokes equations'. We are not writing this to claim that we will solve one of the Millennium Problems, but in order to emphasise that this is an important and recognised topic. Any progress that goes towards its solution is of interest to the community. We believe that our work will go in this direction.