Non-standard parabolic problems in description of biological processes

Systems of reaction-diffusion equations are very powerful tool in description of diffusion and chemical reactions of various molecules inside a certain spatial domain, on the boundary of which we have boundary conditions which usually couple the fluxes (normal derivatives of concentrations) with the values of solutions on the boundary. However, biology provides a lot of examples of phenomena, where this simple form is insufficient and in which both the structure of the system as well as the boundary conditions are different than traditional.

A. Boundary conditions can be expressed by the solutions to a different system, which is defined on the boundary of the considered region. These equations describe certain specific processes taking place on the boundary of the region. In some important situations, the boundary fluxes may depend on the derivatives of solutions with respect to time and possibly also with respect to some spatial derivatives. For example, we can have two systems coupled only by a boundary condition. Moreover, one the system may describe phenomena inside the domain and the other phenomena taking place only at the boundary. Such a situation can happen, when we try to describe the changes of the cell state implied by various external stimuli. Then we have to do with the change of the state of the signaling proteins localized on the membrane of the cell (so called receptors). Such excited receptors may influence the state of intercellular signaling transducers, so called cytosolic kinases.

B. The elements of biological systems can have also an internal structure (internal degrees of freedom). Such a situation takes place for instance if we consider the molecules, e.g. cells which undergo diffusion and posess internal degrees of freedom connected with the concentration of some auxiliary molecules or molecule complexes (cytosolic or mebrane ones), which cannot diffuse freely, because their position in space is determined by the position of their cell-carrier. The mathematical description of such a biological system contains equations which differ significantly from the standard equations with non-degenerate diffusion operator.

C. Still other boundary conditions in description of biological phenomena occur, in which fluxes through the boundary depend on time derivatives of the solution. These are so called dynamic boundary conditions. Direct motivation to investigate this problem lies in the discovery of very fast calcium waves with propagation velocity significantly greater than in case of fast calcium waves known earlier. These experimentally observable waves, called Calcium Induced Calcium Influx (CICI), have velocities up to 1000 $/mu$m/s or even more.

Main goal of the project is an analysis of problems connected to the above described phenomena. Along with mathematical results, it will give an insight into biology of these processes. Moreover, such an analysis can provide information of medical significance. As an example, analysis of atherosclerotic deposits formation and conditions of its development can give us novel information about so called risk factors. We think that our research will be a contribution to cell signaling models and signal transmission from membrane receptors into cell. Analysis of the equations with dynamic boundary conditions will be essential in getting an insight into propagation of very fast calcium waves in biological cells. In particular, we hope that we will be able to answer the following question: Can CICI waves propagate without participation of intercellular calcium stores, i.e. reticulum and mitochondria? Our studies can also explain the process of formation of morphogenetic patterns.