The notion of random group was introduced by Gromov in order to bring sense to questions such as: „how does a typical group look like?" or „what properties are typical for groups"? He defined a random group as a group with the presentation $\langle S|R \rangle$, where S is a set of at least two generators and R is a set of random cyclically reduced words of length l over the alphabet S. In this model, density is a parameter determining the size of set R. When investigating this model we check which group properties hold with probability tending to one as l (the length of words) tends to infinity. A random flag complex is a flag complex such that its 1-skeleton is a random graph. This notion is a higher dimensional generalization of a random graph.

The main goal of the project is to investigate properties of random groups and random flag complexes. Properties that we will analyze are: Property (T) which means that the first cohomologies of a given group with coefficients in each unitary representation vanishes; Haagerup Property, which is a strong negation of Property (T), and vanishing of the first ℓ^2-Betti number, which is the von Neumann dimension of the first group of ℓ^2 cohomologies of the studied group. These notions have found numerous applications in various fields of mathematics, hence deepening of the understanding of these notions will impact modern mathematics profoundly. This constitutes one of our aims in the project and the fundamental task is to provide a better understanding of the behavior of random groups and radom flag complexes.

It turned out that for wide range of densities random groups in the Gromov model have many interesting properties: these are hiperbolic torsionfree and have Property (T) — hence constituted a new class of examples of groups with Property (T). Our goal is to determine for which range of the density parameter random groups in this model have Property (T), Haagerup Property and vanising first ℓ^2-Betti number.

Another model of random groups we will investigate is the square model and the hexagonal model, where we consider a group with the presentation $\langle S|R \rangle$, where R is a set of cyclically reduced words of length 4 or 6, respectively. In the case of these two models, one studies properties that hold for random groups with high probability as the cardinality of the set S tends to infinity. It turned out that the analysis of these models is much simpler than the Gromov model. Therefore, we plan to investigate square and hexagonal model carefully in order to get new information about the theory of random groups in general, and also to test our tools and gain experience before analyzing the Gromov model. Our goal is to determine for which densities random groups in the square model have: Property (T), Haagerup Property and vanishing first ℓ^2-Betti number.

Our last objective is to investigate the properties of fundamental groups of random flag complexes. We intend to determine for which parameters the fundamental group of such complex has a vanishing first ℓ^2-Betti number. We believe that the new tools that we plan to use – such as spectral crietria, modified hypergraphs or mixed square-triangular random group models – will unravel answers to the questions stated above, and thus contribute to the development of the field. Moreover we can compare groups (or complexes) for parameters for which aformentioned properties hold with groups (or complexes) for parameters for which they don't, and identify crucial differences between them. It will be beneficial to understanding of the Property (T), Haagerup Property and the first ℓ^2-Betti number. Analogously, we wish to determine the main obstructions for the random flag complexes to have vanishing first ℓ^2-Betti number.

The notion of a random group can be viewed as group-theoretical concept analogous to random graphs. Random graphs contributed in various branches of mathematics, answering to an abundance of problems. We are convinced that random groups and random flag complexes can play an equally important role, which provides additional motivation for our research.