The theory of dynamical systems is widely used in many branches of modern science. Its methods and tools are used not only to solve strictly technical problems, but also in the analysis of phenomena such as climate change, the behavior of financial markets or processes occurring in the human brain. Typically, the most elementary research in this area is the analysis of stationary points (known also as fixed points) of considered system, i.e. the points in which the speed of changes in the system (the first derivative of displacement) equals zero, and in other words, the system is in an equilibrium state.

Recently, considered theory has been extended with a new type of critical points, known as the perpetual points, which analysis is the subject of research of the proposed project. In contrast to the fixed points, at perpetual points the acceleration (second derivative of the displacement) of the system becomes zero, while the velocity remains nonzero. Perpetual points have been introduced by A. Prasad in 2014. In the original research work their numerous applications have been suggested, e.g. in studies about types of systems (dissipative or conservative) or localizing the hidden attractors. However, both the problem of their interpretation and analysis of their properties still remain open. The aim of this project is to find the answers to various questions related with properties of these points.

The studies will begin with the search of perpetual points in both elementary and complex dynamic models. This will allow to localize and in consequence to analyze the states of the system to which these points lead. These states, named attractors are one of the fundamental concepts of the discussed theory. They represent the stabilized behavior of the system (stabilized dynamics) after a period of temporary disruptions, which are needed to focus the system on a given attractor. From the scientific point of view, one of the most interesting systems are those having many co–existing attractors, named the multistable systems. In such cases, the final state of the model depends on the conditions in which it has been in the initial phase of work. The analysis of such systems is a rapidly developing branch of dynamics, and in our project works we will especially focus on them.

During the work on the project, the impact of the structure of considered models and their parameters on the behavior of perpetual points will be also studied. These points, similarly as fixed points, can appear singly in the system (unique point), as well as co–exist, forming complex structures in the phase space, i.e. the area of system work.

In order to accomplish the project aims, both elementary and complex tools used in studies of dynamical systems will be applied. Numerical algorithms will allow to determine the values of perpetual points existing in the considered systems and the dynamics of the systems themselves. In the case of multistability, the basins of attraction will be used to identify the regions of initial conditions which lead to different attractors. On the other hand, the Poincare maps will allow to determine the dynamics of obtained solutions, which can have a regular (periodic states) and chaotic character. Important from the point of view of the properties of perpetual points will be also to analyze the theory of fixed points, which have been the motivation for the introduction of the former ones.

The reason for considering the research issue which is the subject of the proposed project is the proceeding development of the theory of dynamical systems, which has been mentioned at the beginning. If we establish a thorough analysis of the properties of perpetual points, properly interpret them and consequently understand the mechanism which allows them to work, they may become a new and extremely useful tool in the study of both simple and complex dynamics problems. Regardless of the complexity, the analysis of such problems usually begins from the study of stationary points. Perhaps, in the case of positive results concluding from the research works planned in the proposed project, perpetual points will become another, important element in the investigations of researchers solving problems of dynamics.