DESCRIPTION FOR GENERAL PUBLIC OF THE PROJECT "LÉVY PROCESSES AND COMPACT QUANTUM GROUPS – EXAMPLES, PROPERTIES, CLASSIFICATIONS"

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In the world around us, in the nature, the architecture or the art, we find many symmetric objects, that is the objects that after a natural transformation (of the plane or the space) look the same as before the transformation. For example, if the vertices of the figure are not distinguishable (numbered or colored), we can not distinguish a fixed square from the square rotated for 90 degrees. Also in physics, many of the conservation principles (of energy, of momentum, etc.) can be described as certain symmetries. From mathematical point of view, symmetries of a fixed object form the structure called a group. This means, in particular, that two symmetries performed one after the other is a symmetry again.

In the quantum world, where the Heisenberg uncertainty principle holds, the quantum analogue of the concept of symmetry (the so-called "quantum symmetries") has been sought since long time. One such attempt is the theory of compact quantum groups, created by S.L. Woronowicz in the eighties and nineties of the last century. It turned out to be very rich and interesting also from the mathematical point of view. This theory combines algebraic aspects, which encodes the group structure in the quantum world, with topological aspects that allowed to develop within this theory the whole range of analytical methods. This theory is also one of the 'non-commutative' fields of mathematics, that is it concerns the objects which behaves differently depending on the order of their application. Other such theories, that has also progressed a lot recently, are the noncommutative geometry and the noncommutative probability

Until now, many examples of compact quantum groups has been described – they arise, for instance, as one-parameter deformations of classical Lie groups (the well-known special unitary quantum group $SU_q(2)$) or as modifications of symmetry groups, in which we give up the assumption of commutativity of the generators (this process is called 'liberation' of the group). It happens often that the same quantum group can be described in several ways, each one providing supplementary information about the object. This is exactly the case of two families of quantum groups which I plan to study in details: one of them arises as the deformation of classical groups, while the other comes from the liberation procedure. However, they both can be described by the so-called Tannaka-Krein duality, and in the latter case additional combinatorial structures appear. Is there the any relation between the two families? Can we find for the first family any similar combinatorial structure? Answering these questions will help us better understand the two families of quantum groups and their relationships with noncommutative probability.

Quantum groups are examples noncommutative spaces, on which one can define stochastic processes, and Lévy processes in particular. The latters are processes with independent and stationary increments, which in the classical world have many applications in the description of random phenomena, for example, Brownian motion of a molecule or pricing of financial instruments (e.g. shares on the stock exchange). In the theory of quantum groups no direct applications of Lévy processes to quantum physics is known (yet). It is known, however, that Levy processes on quantum groups contain a lot of information about the groups they 'live' on. One can think about it as if, on the basis of observations of Brownian motion molecules, we could conclude about the size and the curvature of the space the particle is moving in. In my project I'm going to look for specific examples of processes that carry such information. I also want to describe (classify) all Lévy processes for selected quantum groups, completing the current knowledge on the subject. The results obtained in the project will allow to better understand what quantum groups are and what kind of information about them can be found by examining the stochastic processes on them.