UPPER AND LOWER BOUNDS FOR STOCHSTIC PROCESSES

BOUNDS

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A stochastic process is a family of random variables indexed by an arbitrary index set. Intensive investigations of stochastic processes - as fundamental objects in modern probability theory – have born a lot of intriguing mathematical question. Considering a stochastic process one can introduce the notion of a sample path as a function defined on the given index set as the evaluation of the process on a given random element. What is usually investigated in the area of basics of the theory of random processes are properties of sample paths like their shape, their regularity. For example one can think of sample boundedness or sample continuity. The goal is to match for a given sample path property whether a process shares the property or not with a list of computable parameters. The primary objective of the project BOUNDS is to develop modern methods of finding tight bounds for various parameters of sample paths. The project has inherited its name after the title of the seminal monograph by M. Talagrand, which depicts a front line of the study in this field. There are two classes of stochastic processes which are of a foremost importance: Gaussian and Bernoulli processes. In a long line of research on Gaussian processes, initiated in the 1960s, a complete geometric characterization of boundedness of sample paths was finally provided by M. Talagrand in 1987. Soon after, a similar characterization concerning Bernoulli processes has been conjectured. Over the years the problem became popular under the name of "Bernoulli Conjecture" and even a prize of $5000 was funded by M. Talagrand. The question had remained open for nearly 25 years and only recently R. Latała and the leader of the project proved that the Bernoulli Conjecture holds true. This achievement gives an opportunity to understand other classes of stochastic processes such as canonical, infinitely divisible and empirical processes. The agenda of the project BOUNDS is to form a team of experts, PHD students and one eager to cooperate post-doc. The main idea is to use and develop methods like generic chaining, Sudakov minoration, concentration of measure to establish new result and hopefully and push the limits of theory of stochastic processes way beyond the existing knowledge in that respect described in the Talagrand's monography.