

Asymptotic invariants of graded homogeneous sequence of the ideals.

Description for the general public

This research path was initiated in 1974 by by André Galligo. He showed that if we consider a general change of coordinates for ideal I , then we obtain a monomial ideal which consists of leading terms of polynomials, and this ideal is invariant with respect to I . This invariant has been called the *generic initial ideal*, and was denoted by $\text{gin}(I)$. Until Galligo published his work, only some of special cases of the ideals of this type were known.

The subject of generic initial ideal, after publishing Galligo work, was investigated by many mathematician. As a result, some of basics facts about asymptotic invariants of $\text{gin}(I)$ has been published. This topic seems to be well-known analyzed, but recently published articles show that it is still popular and consist of many worth to investigates areas, such as a higher dimensional cases and so on. An important series of articles from last two years deals with geometric interpretation of that ideal and consists of some attempt to give an explicit geometrical description of $\text{gin}(I)$. It is a starting point for that project research. A small group of investigated cases gives a big opportunity to work over not analyzed cases. The first step will be giving a new definition for so-called limiting shape for ideals fulfilling grading condition. That could be helpful in work with mentioned higher dimensional cases and the cases for which the geometry of set of zeros is difficult to describe.

Another interesting field to investigate is to analyze the asymptotic invariants for $\text{gin}(I)$, so the algebraic structure from which we may obtain far-reaching conclusions about structure of the ideal I . As Mark Green showed, generic initial ideals has a lot of invariants. On the other hand, finding that invariants is frequently much easier that finding classical versions of them. A full and thorough investigation of these invariants is main motivation for this project. Another reason is my work with symbolic and ordinary powers in recent years.

In the next phase we want to work with the line arrangements in \mathbb{P}^3 , including famous subject of 27 lines arising as the blow-up of six generic points in \mathbb{P}^2 . It is a basic point to analyze configurations of lines in \mathbb{P}^n , which we are also planning to deal in this project.

The am of this project research is not only to investigate the asymptotic invariants of the graded sequence of ideals, but also Hilbert function or Newton polytope. These object are strong related to generic initial ideals. We want to put main accent on differences between individual invariants.

A totally new way of progress will be to define a new family of powers. The saturation of the ordinary power of the ideal by the maximal ideal. Directly from the definition it is an ideal, but in some point it is more subtle. We encounter here a completely new phenomenons. It is another goal of this project to investigate this case. At the end we want to present a geometrical interpretation using computer tools in these cases in which it will be possible.