MAXIMAL OPERATORS

The main purpose of the planned research is to get a deeper knowledge on the nature of the objects known in the mathematical literature as maximal operators. In order to present the problem in a friendly way, it is worth to start with an analogy. Let us imagine that we want to evaluate fairly, how large is the potential of an athlete practicing particular sport discipline. To perform this task we need to analyze his career and choose the moments in which he was in top form. But what is the top form? By this we mean some large average value of the results that the player got in a certain period. Maximal operators which act on functions on some metric measure space are little more complicated, but the story above actually talks about the genesis of this objects. It was the idea of two British matematicians, Godfrey Harold Hardy and John Edensor Littlewood, the first of which was a big fan of cricket.

There are many versions of maximal operators, including two the most frequently considered, namely the centered and noncentered Hardy–Littlewood maximal operators. We can describe almost strictly the action of the noncentered one as follows. For a given non-negative function f we find its maximal function Mf in such a way that to determine the value of Mf at some point x we calculate the average values that f takes in some areas containing x and then choose the largest of them. Referring to an earlier analogy, we could say that this corresponds to a situation in which a sports critic looks at the player favorably and evaluates his form at the moment as high as possible according to its working definition. In addition, let us explain that the mentioned areas are topologically just balls. The concept of balls has in topology a broader meaning, but in the classical situation when we work with the three-dimensional Euclidean space, it is the same as usual balls in stereometry. The only difference in defining the centered maximal operators is that the balls which are taking into account must have a center at the point x.

The theory of maximal operators has found many applications in various fields of mathematics, especially in harmonic analysis. Using maximal functions one can prove several classic Lebesgue differentiation theorems. Moreover, the nature of this objects allows to estimate from above the precise values of certain other mathematical quantities, for example some operators describing the heat distribution in time and space. From the point of view of potential applications in other areas of mathematics the key is to know how large can be Mf compared with the initial f or more generally, what properties of Mf are a consequence of the respective properties of f. This problem has been already solved in the most typical cases, and the obtained results belong to the canon of harmonic analysis. However, there are still not fully explored aspects of this subject. This project involves looking at some unique properties of maximal operators and describing the conditions in which they may occur. More precisely, we are focused on the functions f belonging to some particular function space, for example some Lebesgue or Lorentz space, and we try to answer whether and when the functions Mf may belong to the same or other space with the size, expressed in terms of the approptiate function space, comparable to the size of f. The introduced class of the considered metric measure spaces should allow to make the associated maximal operators so that it could be possible to extract some of their characteristic elements that may appear only under certain specific conditions.