Abstract (for the general public):

Symbolic computations on first-order definable objects

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Objectives. The proposed project belongs to mathematical foundations of computer science. The principal objective is to lift algorithmics from finite input objects to infinite objects which are finitely described by first-order formulas (first-order definable). The approach one has to apply in order to computationally manipulate infinite objects is forcedly symbolic, i.e. algorithms must process representations of input objects, instead of the objects themselves.

Illustrating example. To give an idea of our setting, consider the following decision problem: given a graph G, decide whether the graph is *planar* (i.e. whether the graph can be drawn on the two-dimensional plane)? Due to the famous Kuratowski's theorem, planarity admits a characterization by means of forbidden patterns: a graph is planar if, and only if, it does not admit a clique K_5 nor a bipartite graph $K_{3,3}$ as a topological minor. In consequence, the decision problem is decidable in polynomial time.

Can one solve algorithmically the planarity problem in case when the input graph is infinite, but defined using first-order logic with, say, equality? Here is an example of such graph: its nodes are pairs of natural numbers $V = \mathbb{N} \times \mathbb{N}$, and its edges $E \subseteq V \times V$ are defined by the following formula

$$(m,n)E(m',n') \iff n = m' \ \vee \ (m = n' \ \wedge \ n = m').$$

Indeed, it turns out that planarity is decidable for so defined graphs, due to the following three observations. First, Kuratowski's theorem still holds for infinite graphs; therefore the decision procedure needs to check that either K_5 or $K_{3,3}$ is a topological minor. Second, the question whether a fixed finite graph is a topological minor of a first-order definable one is reducible to first-order satisfiability. And third, satisfiability of first-order logic with equality is decidable.

Motivation. Motivation for the research comes for various sources in computer science where computations involving infinite *data domain* are investigated. The data values may represent time, names, content of an XML document, etc.