

Our project will develop the mathematical field of Algebraic Topology in order to find applications other fields such as Robotics, Number theory and Social Choice.

In order for a robot to function properly it must be given instructions. This means that it should be told what to do on every situation it encounters and it should also be told how to perform tasks. These instructions are called *motion planning algorithms*. Every functioning robot must have a well thought motion planning algorithm.

Nowadays one of the main priorities in design is minimality. This means that machines should be small, light, fast, energy efficient, and cheap!

A too complicated motion planning algorithm often results on violating these principles as it may need lots of memory and processor power. These needs may actually drastically influence the robot's design! This specially applies to microscopic robots as adding an extra chip, for instance, may cause the design to physically fail.

Because of this, it is important to study how complex (or simple) a motion planning algorithm can be. This can be described by a number called the *topological complexity* of a robot. This number measures the best degree of simplicity we can hope for when designing a motion planning algorithm for the robot.

As a very simple example, consider a robot lying on an empty room. We can give him the following instruction "to go from point A to point B, follow the straight line connecting A to B". This is a very efficient and simple motion planning algorithm since it requires almost no *thinking*. Now, let us put an obstacle in the room, or even two. Then previous motion planner would not work (unless we want the robot to crash!). One must then give it extra instructions such as "if an obstacle is on the way, go around it clockwise". This new algorithm is more complex as the robot has to *think* whether an object is on the way or not. It may also be less efficient as going around the object counter-clockwise might be shorter. The topological complexity of the first robot is very *low* as for the topological complexity of the second one is *higher*.

It turns out that topological complexity depends only on the *topology* of the robot. Its study can be thus done through purely mathematical methods. There are many pure mathematicians working on these problems using topological techniques and, as consequence, special relations with other branches of mathematics have been found.

A second situation in which this concept of complexity arises is in Social Choice Theory. Suppose that a certain number of agents, say AI's, have to vote on where to build a dock on a lake (or an island). Then all the possible choices have the *topological form* of a circle. The problem of reaching an actual decision is related to the existence of a *mean* or *average* function on the circle. We will study ways to find these functions on much more complicated spaces than a circle and how likely they are to be found.

A third scope of action of our project is the ancient and fascinating mathematical field of Number Theory. This field studies the relations existing among numbers. Surprisingly, our topological methods actually give information on a very important class of numbers called the Bernoulli numbers.

Through the course of history, the development of mathematical theories have unexpectedly found applications in other fields of mathematics and science, even many years after they were created. On the other hand, attempts to solve real-world problems have given birth to beautiful mathematics that turn out to be much more important than the initial problem they were meant to solve. This project is another example of this amazing interaction existing between Pure and Applied Mathematics.