Differential operators over noncommutative algebras.

Exploration of spaces is conducted in various ways - we can study for example their topological properties (for example, distinguishing between the sphere and the torus), their shapes and sizes (distinguishing a cube from a sphere and a big sphere from a small one). One of the effective methods of research on spaces is the analysis of possible types of functions and operations on them. A special role is played then the operations that meet certain criteria that correspond to differentiation. The so-defined differential operators are the basis of modern differential geometry, mathematical physics and the theory of differential equations, including many equations describing the relevant physical processes.

In the description of the world one does not necessarily use the space and there is a large class of objects that only correspond to the spaces. The classic example is the phase space of quantum mechanics, where the position and momenta of the objects separately correspond to functions on ordinary spaces, but together form the "noncommutative space" described only as a noncommutative algebra.

Noncommutative geometry is the study of these objects, and one of its ambitious goals is to describe and study such algebras ("noncommutative spaces") by geometrical methods, in the same way that (differential) geometry is the study of spaces. From the point of view of differential geometry, algebra chosen should correspond to the algebra of smooth functions – so geometry investigates regularities of functions, the shape and size of the space the live on.

The proposed project has as main objective to examine the applicability of the definition proposed recently by the author and co-investigators, of the differential operators of the first order for noncommutative geometry. Analysis of this definition and its application and its consequences for certain topological structures will be a key aim of the proposed research.

All of these goals are part of a larger problem, namely the definition of a sufficiently broad class of differential operators for noncommutative algebras. The current definition is either too wide (that is, allow multiple operators, which are rather pseudodifferential or too narrow (in interesting cases there are no operators which fulfil the conditions). We hope that initiated research is a novel one that will effectively develop methods of differential studies for noncommutative algebras, which extend the topological methods.

The recently proposed construction is new, fresh idea, which brings the hope of significant advances in the understanding and use of models noncommutative geometry both to study algebras and their invariants as well as in other applications. In the short term it seems most important to verify whether the compact quantum groups, which are noncommutative generalizations of symmetries behave in a similar way as their classical counterpart, compact Lie groups. The primary application in other fields of research is the ability to use an expanded definition of the structure for the physical models to describe the fundamental interactions.