A quest for new computer logics

Numerous objects appearing in various areas of computer science, including software and hardware verification, knowledge representation and databases, can be modelled as relational logical structures, i.e., sets of elements, on which some mathematical relations are defined. We would like to perform some reasoning tasks concerning such models, e.g., we may want to check if a given program does indeed what it is intended to do, if a given knowledge base is consistent, or if it implies a given fact. To this end we can specify properties of the considered objects by means of logical formulas and then perform some automated analysis. Very often such analysis boils down to verifying *satisfiability* of a given formula, i.e., checking if it has a model.

To perform the procedure describe above we need an appropriate language, in which properties of the objects we are interested in can be described. Unfortunately, in the case of probably the most natural specification language, first-order logic, its satisfiability problem is undecidable, which means that there is no algorithm which (even in theory) could solve it. Thus, a constant effort of the international research community is taken to look for languages having good properties, i.e., a decidable satisfiability problem (possibly having additionally a low computational complexity) and a relatively good expressive power. Of course, there is a natural trade-off: the more we can express in a given language, the harder it is to reason automatically in it.

A status close to an industrial standard has been acquired by modal, temporal and description logic, which are even sometimes called *computer logics*. Formally, they are not fragments of first-order logic, have their own specific and not always very friendly syntax, but usually can be translated into some fragments of first-order logic. Moreover, the images of such translations fit usually in small fragments of first-order logic. A few elegant such fragments motivated by practical applications are known. Investigating them can help to better understand the properties of standard computer logics, to identify ingredients, which can be safely added to them, and, complementarily, ingredients or their combinations which lead to increase of the computational complexity. Finally, such works may lead to discovering formalisms, which will become new computer logics themselves.

The main goal of this project is to investigate the decidability and computational complexity of the satisfiability problem and some fundamental related problems motivated by practical applications of some extensions of a few fragments of first-order logic: uniform one-dimensional fragment, UF_1, and the two-variable fragment, FO^2, the unary negation fragment, UF_1, and fragments with guarded negation, GNF, and guarded quantification, GF. Among the fundamental extensions of the above basic formalisms are equivalence and transitive relations, linear orders and counting quantifiers, all of which are very natural in practical applications. We also plan to examine satisfiability of the mentioned logics in some important classes of structures, such as words and trees. Among the mentioned formalisms, FO^2 and GF are pretty well understood, but still there are some interesting questions left open which we want to answer. The main tasks of this project will however concern the logics which have been proposed during the last few years: UNF, GNF and UF_1.