SELECTED TOPICS ON THE BORDERLINE OF NONLINEAR PDES AND GEOMETRIC MEASURE THEORY

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DESCRIPTION FOR THE GENERAL PUBLIC

The aim of this project is to investigate nonlinear partial differential equations that appear on the borderline of geometry and of the calculus of variations. This includes proving new theorems on the existence and regularity of solutions, and studying the properties of particular solutions and whole classes of solutions, with particular emphasis on their *singularities*.

The calculus of variations itself is one of the branches of modern mathematics that have ancient origins. The solution of the isoperimetric problem – ascertaining that among all planar domains with a fixed perimeter the circle has the largest area – was alluded to in the mythical story of Dido, the founder of Carthage. Circular shapes of numerous medieval cities can be observed in the famous 16th century *Civitates orbium terrarum* of Braun and Hogenberg not because of some particular charm of the circle but simply because that shape yields the minimal cost of construction of the city walls, protecting a given area of land. Similarly, the roof of the Olympic Stadium in Munich has the shape of a minimal surface not only because its designer was an eccentric[1] but also because such a design yields the minimal mass of the roof having a fixed contour.

Numerous other problems of the calculus of variations are studied by physicists that look for stable equilibria of objects whose shape or state can be described not just by one or two parameters but by (potentially) infinitely many parameters – to wit, as a graph of an arbitrary function of one or several variables. It is worth noting that many problems that we would like to describe by continuous, or even smooth functions – with graphs, curves, surfaces that have no gaps or kinks or corners and crossings – do have *singularities*: the nature shows us self–intersecting soap films, defects of liquid crystals and vortices in fluids or superconducting materials. The description of the structure, size and position of such defects of solutions, and the questions about their stability or lack thereof (or about conditions that guarantee the absence of defects), lead to difficult purely mathematical problems on the borderline of modern geometry and analysis, and require a mixture of various technical methods and tools.

Multidimensional variational problems lead to partial differential equations (PDEs); the solutions are described as vector–valued functions of several variables. Due to natural constraints of geometric or physical origin the equations are nonlinear, which makes them more difficult to study or to solve. A vast part of the progress in the theory of PDEs that arise on the borderline of physics and geometry is connected to the creation of the theory of distributions and to the applications of nonlinear functional analysis. Despite many achievements and the work of numerous mathematicians, this domain still contains lots of open problems.

The present project has purely theoretical and coginitive character but – in a distant perspective – it is driven and motivated by the significance of the calculus of variations and of the theory of PDEs, which is due (also) to the applications of mathematics to the description of the optimal form and optimal shapes. The author's main aim is to study the stability and instability – with respect to changes of boundary conditions – of the singular sets of solutions of equations that appear e.g. in the Ericksen's model of liquid crystals and other, mathematically related, problems. We also plan to work on new methods and tools than can be applied to such problems. The research will be conducted with young collaborators, PhD and MSc students from the University of Warsaw.

[1] Frei Otto, 1925–2015, was famous for his use of lighweight structures.