

Dynamical systems are often subject to random influences, such as external fluctuations, fluctuating initial conditions, and uncertain parameters. The proposed project is devoted to ergodic properties of such dynamical systems. The understanding of their ergodic properties is important for purely mathematical reasons as well as for possible applications in statistical physics, biology, chemistry and many other fields.

The evolution in time of a random system is described by stochastic processes. In the case when the behaviour of a system in the future depends on the current state but is independent of the past we say about Markov processes or Markov chains. The theory of Markov processes/chains has been extensively developed for many years. One of the most important questions in this theory is the question about the existence of an invariant measure and its stability, i.e. other initial measures are attracted to it as time evolves. The classical example is the Boltzmann equation and the celebrated H-theorem, which says that the process evolves to the state described by the distribution that maximalize the entropy.

As an appropriate model for our description may serve an iterated function system. The iterated function system consists of a set of transformations and some probabilistic vector. Having such systems we may observe the evolution of initial states due to action of randomly chosen transformations. We may ask about limit behaviours of the corresponding Markov chain. There are still open questions for iterated function systems consisting of homeomorphisms defined on the simplest closed manifold - the circle. The most urgent question is devoted to the rate of convergence for stable iterated function systems. Iterated function systems have been completely analysed in the case when the system is contractive or at least contractive on average but this assumption fails to be satisfied by homeomorphisms on the circle. We would also like to examine the behaviour of function systems in more general setting (e.g. action of Möbius maps on the Riemann sphere, in particular Kleinian and Fuchsian groups).

The second important question, closely related to the previous one, asks about the geometrical structure of invariant measures. In particular, we are interested in whether the invariant measure has density or is singular with respect to Lebesgue measure.

In the proposed project we consider the asymptotic behaviour of more complex dynamical processes including Markov processes corresponding to stochastic differential equations. Similar questions as for iterated function systems are posed in the proposal. In particular, we would like to obtain sufficient conditions for central limit theorem, law of the iterated logarithm and law of large numbers for a broad class of stochastic processes. Additionally we would like to extend the lower bound technique originated in the papers by A. Lasota and J. Yorke and show that it provides the tool for studying the behaviour of asymptotically periodic Markov processes.