# Description for the general public of the research project

Quantum mechanics forced the scientists to develop new methods of describing the nature. It turned out that the appropriate language is provided by the so-called operator theory. However, a proper object describing a given quantum system is not a single operator but a whole family of these; such structures are called von Neumann algebras (named after John von Neumann, one of the founders of mathematical description of quantum mechanics).

Nowadays we know a whole variety of different von Neumann algebras – some of them very simple and some of them extremely complicated. For many years only the easiest examples were needed in the mathematical description of quantum mechanics, the other ones being interesting, for various reasons, only to mathematicians. However, ever since the advent of quantum field theory (counterpart of quantum mechanics, in which one allows the number of particles to be infinite) the situation changed drastically – the hitherto used tools turned out to be insufficient. Mathematical physicists who tried to extend the methods using von Neumann algebras realised that in this case the most complicated ones are inevitable – the so-called algebras of type III. Even though this has not yet given a satisfactory description of quantum field theory, it gave a solid motivation for studying von Neumann algebras of type III, which will be the main goal of our project.

We have already said that von Neumann algebras are families of operators. Yet not all of the elements of of such a family are equally easy to deal with – some are fairly simple, while the structure of the other ones might be very intricate. We are aiming at investigating how we can approximate the more complicated elements by the ones that are easier to study. The analogue of this situation in classical geometry is the approximation of a circle by inscribed polygons, with a growing number of sides. There is, however, a natural question to ask: what does it mean that the circle is well approximated by polygons? After we decide on the method of measuring the accuracy of approximation, we may legitimately ask, how many sides we need to obtain a satisfactory approximation. We plan to deal with similar questions in our project.

For another topic of our study we chose analogues of random processes. Classical examples are: sequence of coin tosses or Brownian motion, i.e. chaotic movement of pollen grains in a liquid. Since the quantum mechanical description of nature is very much different from the one offered by classical physics, the description of random phenomena also changes. We will be interested in random processes defined in the language of von Neumann algebras. These will include the analogues of coin tosses and Brownian motion but we will not restrict our attention to these examples.

The main goal of our project is to accelerate the research in the theory of von Neumann algebras of type III. We believe that gradually new connections to other areas of mathematics will be revealed, which may lead to more interdisciplinary projects in the future.