DESCRIPTION FOR THE GENERAL PUBLIC

- **State the objective of the project.** The objective of the project is to understand to what extent the form of probabilistic conditional structures, e.g. regression functions, or independence assumptions, influence properties of objects which are important in probabilistic and statistical modeling. Often such issues are related to characterization problems of probability distributions. In recent years more and more often, due to complexity of models, these distributions live on abstract mathematical objects, e.g. symmetric or homogeneous cones or function spaces. In particular, for stochastic processes we will be interested in connections between ASEPs (asymmetric simple exclusion processes) and quadratic harnesses (processes defined through their linear conditional expectations and quadratic conditional variances) and in properties of multivariate quantum Bessel process as well as hypergroup methods of Markov processes construction. We will be also concerned with characterization problems for random matrices and their generalizations (random elements in symmetric and homogeneous cones) such as generalizations of the Lukacs property, the Matsumoto-Yora property and a new independence property for the Kummer and gamma distributions. These problems are related to solutions of some special functional equations, e.g. the Cauchy or Olkin-Baker equations on cone structures with suitably defined multiplication operation. We would also like to continue investigations of analogy in characterization results which is more and more widely observed between the classical and free probability. In this respect we will pursue new connections to the operator-free probability. We will be also interested in developing the theory of discrete Bayesian graphical models through a definition of suitable class of priors. It will be realized through introducing sufficiently rich classes of distributions related to graph structures. We plan to extend the fundamental hyper-Dirichlet distribution to other models than those generated by decomposable graphs. We also plan to search for characterizations of such newly defined distributions through natural conditions of local and global independence of parameters.
- **Describe the research to be carried out.** Particular investigations to be carried on in this project are: applications and generalizations of the representation formula for the ASEP which is based on moments of QH processes; connections between birth and death processes and the abstract multivariate quantum Bessel process; application of the hypergroup theory to construction of semigroup of transitions kernels for Markov processes; extensions of Lukacs and Matsumoto-Yor characterizations to homogeneous cones (an example of such a cone are matrices of given dimension with prescribed zeros); matrix version of Kummer and gamma laws independence characterization; free and operator-free versions of classical regression characterizations; a search for a priori distributions for discrete Bayesian model, in particular, we have in mind a generalization of the hyper-Dirichlet distribution to Markov equivalent models based on, so called, essential graphs.
- **Present reasons for choosing the research topic.** The theme of characterizations is interesting for both theoretical mathematics and applications. It often allows for a unique and simple description of complicated probability systems. We think that algebraic properties of QH processes, which are responsible for connections with the ASEPs, are of special mathematical elegance. The investigation of connections between the multivariate quantum Bessel process with the birth and death processes is a natural consequence of Biane's observation for univariate processes. It is also a natural question to seek for characteristic properties of important distributions which naturally arise as analogues of classical distributions when we pass from a univariate setting to more complicated multivariate structures, e.g. to algebraic cones. The research of a priori distributions in discrete Bayesian graphical models will help to create sufficiently reach family of such distributions. Probability measures of this type (in particular, the celebrated hyper-Dirichlet distribution, are known mostly for decomposable models. To close this gap we plan to study a version of hyper Dirichlet distribution for wider class of models based on chain graphs.