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RANDOM GREEDY ALGORITHMS FOR HYPERGRAPH COLORING

Description for general public

Suppose there is a set of elements that we want to split into two groups. A real-life example can be a database stored in two data centers, or a computer network with two separate power sources.

We are given, however, some constraints for such a split. Specifically, some subsets of elements are forbidden to be all put into the same group. For example, the computers mirroring the same dataset should not, due to safety reasons, be powered from the same source. Such settings are modeled as a *hypergraph 2-coloring* problem. The elements are the *vertices* of hypergraph, and the forbidden subsets are the *edges*. The split corresponds to an assignment of one of two colors to each element. The graph is said to have the *property B* if the desired coloring is possible.

A natural question arises, stated first by Erdős and Hajnal in 1961:

What is the minimal possible size (i.e. the least number of edges) of a hypergraph whose every edge contains exactly k elements and which does not have the property B?

Despite a relatively simple statement, this question still remains to be fully answered. It is known that this number (called $m(k)$) is at most $c_1 \cdot k^2 \cdot 2^k$, and at least $c_2 \cdot \sqrt{\frac{k}{\log k}} \cdot 2^k$, where c_1 and c_2 are some fixed constants. The exact order of magnitude of $m(k)$ is still unknown, as well as any constructive methods of obtaining small hypergraphs without property B.

In our work, we plan to search for better bounds on $m(k)$, as well as for the similarly defined parameter $\Delta(k)$ – the least integer such that if every edge has common elements with at most $\Delta(k)$ other edges, then the hypergraph has the property B. To these problems, we intend to apply new variants of *random coloring* method along with the analysis of *greedy algorithms*. This technique, simply put, provides a probabilistic analysis of a coloring obtained by a procedure that randomly assigns elements to groups, and then repeatedly improves the assignment.