1 Research project objectives

For many important optimization questions there are no known efficient algorithms solving the problem exactly. One way of coping with this situation is using approximation algorithms i.e. algorithms finding a solution that cannot be worse than α times the optimal solution, where α is an algorithm-specific ratio. A next obstacle in practical applications is that one often has to model information about incomplete data. It is common in case a company wants to make an investment but it cannot fully predict future customers' behavior. Another example is managing a web portal which must be ready to serve all the clients' requests when having access only to statistics about the network traffic.

The goal of the project is to extend results from the classic theory of approximation algorithms to make them work in situations when a part of data is described only with a probabilistic distribution. Some results for probabilistic models are known but they often require strong assumptions i.e. independence of events. We would like to get rid of these assumptions and develop a theory working in a relatively simple model allowing arbitrary probability distributions.

2 Considered problems

Studied optimization problems have aroused interest in the theoretical computer science since the 70s. Here are some of them.

- Facility Location: We want to open several facilities. It is known where potential buyers are located and how much it would cost to build a facility in particular places. Where should we build facilities in order to minimize the total cost of construction and commodity transport?
- Steiner Tree: We are given a network e.g. describing telecommunication connections. We would like to buy a cheapest subset of connections that guarantees connectivity between some important terminals. Steiner Tree can be also used to model problems in circuit design and computational biology.
- Job Scheduling: We are given a set of machines and a list of tasks to execute. Each machine can run only one task at once. The tasks may differ in difficulty level and dependencies between each other.
- Auction Design: Our aim is to sell a collection of goods according to some rules e.g. we must not withdraw a submitted offer. We have partial knowledge about which buyers are interested in our goods and how much they are willing to pay. How should we design the auction in order to maximize the revenue?

In our model we assume that a part of data is initially unknown and we have to make all the decisions knowing only the probabilistic distribution. The aim is to optimize the expected value of a solution. In aforementioned problems we could get incomplete information about a set of clients interested in our goods (Facility Location, Auction Design), a set of terminals to connect (Steiner Tree), or a set of tasks to execute (Job Scheduling).

Beside the construction of algorithms, we plan to understand how the structure of a problem changes when data turns nondeterministic e.g. what properties the optimal solutions have.

3 Expected impact of the research project

The main motivation of the research is based on the practical applications of the studied optimization problems. We expect that allowing algorithms to run on incomplete data would lead to broader usage of algorithmic techniques in industrial applications and might reduce the gap between the theory of algorithms and practice. What is more, some issues necessarily require stochastic approach e.g. online dating systems or kidney exchange programs.

From the point of view of the theoretical computer science, positive results in the project could help unifying the theory of stochastic optimization. Whereas the previous works in this area used specific techniques working in different models, our approach forms a coherent theory in which we have already succeeded in obtaining preliminary results for some important optimization problems.