

Geometry of Banach spaces in analysis of high-dimensional numerical problems

POPULAR DESCRIPTION OF THE PROJECT

For almost twenty years the 'reduced basis' method is extensively used in numerical solving of parametric partial differential equations. Since equations of this kind are a fundamental tool to solve many practical problems, it turned out to be very useful in various applications. Algorithms which were developed in the framework of this method are applied also in many other parametric problems, not only to look for a solution for a given parameter but also in other questions e.g. to find a solution with special properties.

Investigation of mathematical foundations of 'reduced basis' method and the algorithms created for its application was initiated only few years ago. Many arising problems can be formulated in the language of functional analysis, approximation theory and geometry of Banach spaces. The project aims at the study of problems which very generally can be described as study of the size of various sets which arise while using the reduced basis method. As a measure of 'size' I plan to use classical tools of approximation theory, in particular various widths.

This circle of problems is important for at least two reasons. Solutions to this type of problems allow us to evaluate when it is advantageous to use a given algorithm or the whole reduced basis method and when it is very costly or even impossible. We can also infer if the algorithms we have are close to optimal; maybe we should look for better. Secondly those problems introduce questions of new type to functional analysis, approximation theory or geometry of Banach spaces.