The aim of the project is to obtain progress in the three following problems of additive combinatorics:

1. Polylogarithmic lower bounds in Littlewood problem in $\mathbb{Z}/p\mathbb{Z}$,
2. Improved upper bound for weak B_k subsets of $[n]$,
3. The degree of regularity of the equation $x_1 + \cdots + x_k = y_1 + \cdots + y_k + b$.

The Littlewood problem belongs to classical questions in additive combinatorics. Green and Konyagin conjectured that if $A \subseteq \mathbb{Z}/p\mathbb{Z}$, $|A| < p/2$, then $\sum_r |\widehat{1}_A(r)| \gg \log |A|$. Analogous conjecture (stated by Littlewood) for finite subsets of $\mathbb{Z}$ was confirmed by in 80's independently by McCgee, Pigno, Smith oraz Konyagin. In a sequence of papers Green, Konyagin, Shkredov and Sanders showed polylogarithmic lower bounds on $\sum_r |\widehat{1}_A(r)|$ for all subsets of $\mathbb{Z}/p\mathbb{Z}$ except for "medium size" sets roughly of size $p/\log^C p$, where C is an arbitrary constant and $p > p_C$. The main objective of the project is to prove a polylogarithmic lower bounds on $\sum_r |\widehat{1}_A(r)|$ for "medium size" sets.

A set $A \subseteq [N]$ is called weak B_k if it does not contain any solution to the equation $x_1 + \cdots + x_k = y_1 + \cdots + y_k$ in distinct elements $x_i, y_i \in A$. Ruzsa proved that $|A| \leqslant (1 + o(1))k^{2-1/k}N^{1/k}$. The next objective of the project is to show that $|A| \ll k^{2-c}N^{1/k}$, for some constant $c > 0$.

There exist partitions $\mathbb{N} = A_1 \cup \cdots \cup A_r$ such that each set A_i not containing any solution to the equation $x_1 + \cdots + x_k = y_1 + \cdots + y_k + b$, provided that $b \neq 0$. The smallest r with the above property is called the degree of regularity of a given equation. Fox i Kleitman conjectured that for each k there exists b such that the degree of regularity of the equation $x_1 + \cdots + x_k = y_1 + \cdots + y_k + b$ equals $(2 + o(1))k$. The last objective of the project is confirm this conjecture.

The reason of choosing the above topic are their importance and potential applications of results obtained in the project. New results concerning these problems attracts attention of specialists in the field.