Summary of the project: **Noncommutative probability with applications.**

The main objective of the project is the contruction and study of new generalized noncommutative stochastic processes, in particular the *Generalized Brownian Motions* (GBM), associated with deformed commutation relations, their relations with classical probability and study of the related von Neumann algebras.

Classical stochastic processes describe various phenomena, where random factors appear, and are based on the notion of classical independence of random variables. In the noncommutative probability the random variables do not commute, since they are self-adjoint operators on a Hilbert space (e.g. matrices). Moreover, in noncommutative probability the notions of independence are different from the classical one, and these are boolean, monotonic, free, conditionally free and bm-independences. The notion of distribution and expectation of a random variable are also different. In general, noncommutative probability space is a unital $*$-algebra $\mathcal{A}$ with a state φ, which plays the role of expectation. The distribution of a random variable $A = A^* \in \mathcal{A}$ is a probability measure μ_A on the real line $\mathbb{R}$, such that the function $\mathbb{C} \ni z \mapsto \varphi((z-a)^{-1}) = \int_{\mathbb{R}} \frac{\mu(dx)}{z-x} := G_\mu(z)$ is the Cauchy transform of the measure. The Cauchy transform palys the role of the Fourier transform in classical probability and is the main analytical tool. For defining noncommutative stochastic processes (gaussian, Poisson, Wiener or Lévy) one generalizes the abstract properties of the classical counterpart. For example, the gaussian measure is the measure obtained in the central limit theorem for given independence – it is the Wigner law for the free case and the arcsine law for the monotonic case. The gaussian operators are constructed as the sum of the operators of creation and annihilation by vectors (on a deformed Fock space), and their distributions are the gaussian measures. The Brownian motion is obtained in the same way, when one considers the specific vectors of the form $\chi_{[0,t]}$ for $t > 0$. Other noncommutative stochastic processes one defines in asimilar way, adding proper modifications (e.g. related to the number operator). In general Lévy process is stationary and with independent increments (for the specific independence). They consists of self-adjoint operators, so one can consider the algebras they generate, in particular their bicommutant, which is the associated von Neumann algebra.

How can one construct deformed Fock spaces, on which the creation-annihilation operators act? The idea is as follows. If a contraction T on $\mathcal{H} \otimes \mathcal{H}$ is given, then one can extend it to $\mathcal{H}^{\otimes n}$ by the formula $T_k := I^{\otimes(k-1)} \otimes T \otimes I^{\otimes(n-k-1)}$ for all $1 \le k \le n-1$. If T satisfies the Hecke condition: $T^2 = (q^2-1)T + q^2 I$ for some $0 \le g \le 1$ and the Yang-Baxter conditions $T_k T_{k+1} T_k = T_{k+1} T_k T_{k+1}$ for $k \in \mathbb{N}$ and $T_k T_j = T_j T_k$ if $|k-j| \ge 2$, then with this T one can construct deformed symmetrization operators P_n^T on the full Fock space. These operators allow to define new structure (scalar product) of deformed Fock space and associated (T-deformed) operators: creation $a_T^+(f)$ and annihilation $a_T(f)$ byvectors $f \in \mathcal{H}$. The commutation relations for these creation and annihilation operators are the deformed commutation relations. The gaussian operators are $g_T(f) = a_T^+(f) + a_T(f)$, and if $\mathcal{H} = L^2(\mathbb{R}, d\mu)$ then for $t > 0$ with $f_t := \chi_{[0,t]}$ being the indicator function of the interval $[0,t]$, the operators $g_T(f_t)$, define the generalized Brownian motion. Construction of this type and studying their properties are the main objectives of the project.

Moreover in the Project we shall study the deformations of operators $A \mapsto W(A)$ for which the distribution of $W(A)$ is a known transformation of the distribution of A, like t-transformation or U-transformation. For these deformations we shall investigate their influence on the behaviour of the spectra of operators (eigenvalues of matrices). We shall also study models of mixed independences (boolean-monotonic, boolean-free, classical-free), related limit theorems (central or Poisson) and related combinatorics of partitions as well as geometrical properties of the positive cones associated with these independencies, and also related noncommutative stochastic processes (gaussian or more general Lévu processes).

The topics to be studied in the project are within the main streams in the noncommutative probability. In particular it will be of great importance to construct and investigate new models of noncommutative stochastic processes and associated von Neumann algebras. As we expect the results of the Project should provide new type III factors, which might be associated with the famous Kadison problem of isomorphism of free group factors with different numbers of free generators, as well as reveal new phenomena and give new boosts in the study of operator deformations and new models of noncommutative probability.