

## The study of tail asymptotics of solutions to random affine equations

This project is dedicated to the study of asymptotic tails of the solutions to random affine equations. These are the stochastic equations of the form  $R \stackrel{d}{=} MR + Q$  (equality in distribution), where  $R$  and  $(M, Q)$  are independent on the right side of this equation. Such equations naturally arise in many areas of applied mathematics, eg. in economics, physics, biology or sociology. Unique solution  $R$  of this equation is called the perpetuity and is the main subject of interest. Both  $M$  and  $Q$  are subject to random fluctuations. These are the distributions of  $M$  and  $Q$  and the interactions between them that impact on the randomness nature of a perpetuity.

Perpetuity can be interpreted as follows: if  $Q$  represents the random payment and  $M$  the random discount factor, then  $R$  is the present value of a commitment to pay the value of  $Q$  every year in the future. Because of this simple and very natural form, perpetuities have attracted a well-deserved interest in applied mathematics. They were used in the study the brightness of the Milky Way, pollution models, financial investment models, radioactivity models, random walks or even models of cultural inheritance. It should be emphasized that an intensive research on this field is caused not only due to their natural applications, but also from the fact that seemingly simple problems in this field require the use of advanced mathematics.

One of such problems is to find a distribution of random variable  $R$  for given  $M$  and  $Q$ . It turns out that this is possible only in a few cases, which is a drawback of applicability of this theory. However, in applications very often it is not necessary to know the whole distribution of a perpetuity. Sometimes, the knowledge about the probability that  $R$  is very large (or very small) is enough for applications. In particular, if we  $R$  interpret as value of the annuity, it is important to know what is the probability that the present value of the annuity  $R$  will exceed given threshold  $x$ , that is,  $\mathbb{P}(R > x)$ . The main subject of research on random affine equations is dedicated to the study of tail asymptotic. Theorems with which we will deal during the project can be understood as follows: for given  $M$  and  $Q$  we search for a special function  $f$ , so that for large  $x$ , the (unknown) value of  $\mathbb{P}(R > x)$  is close to the (known) value of  $f(x)$ . Looking for the tail asymptotic of a perpetuity is equivalent to looking for such function  $f$ .

There exists a vast literature regarding results on the tails asymptotic of a perpetuity when  $M$  and  $Q$  satisfy certain conditions. Asymptotics however, are not known if these conditions are not satisfied. The aim of this project is to give full description of the tail asymptotics of a perpetuity for any  $M$  and  $Q$ .